Question	Scheme	Marks	AOs
7(a)	$(1+t)\dfrac{\mathrm{d}P}{\mathrm{d}t}+P=t^{\frac{1}{2}}(1+t)\Rightarrow\dfrac{\mathrm{d}P}{\mathrm{d}t}+\dfrac{P}{1+t}=t^{\frac{1}{2}}$	B1	1.1b
	$I=\mathrm{e}^{\int\frac{1}{1+t}\mathrm{d}t}=1+t\Rightarrow P(1+t)=\int t^{\frac{1}{2}}(1+t)\,\mathrm{d}t=...$	M1	3.1b
	$P(1+t)=\dfrac{2}{3}t^{\frac{3}{2}}+\dfrac{2}{5}t^{\frac{5}{2}}+c$	A1	1.1b
	$t=0,\ P=5\Rightarrow c=5$	M1	3.4
	$P=\dfrac{\frac{2}{3}t^{\frac{3}{2}}+\frac{2}{5}t^{\frac{5}{2}}+5}{(1+t)}=\dfrac{\frac{2}{3}8^{\frac{3}{2}}+\frac{2}{5}8^{\frac{5}{2}}+5}{9}=...$	M1	1.1b
	= 10 277 bacteria (allow awrt 10 300)	A1	2.2b
		(6)	
(b)	$P=\dfrac{\frac{2}{3}t^{\frac{3}{2}}+\frac{2}{5}t^{\frac{5}{2}}+5}{(1+t)}\Rightarrow\dfrac{\mathrm{d}P}{\mathrm{d}t}=\dfrac{(1+t)\left(t^{\frac{1}{2}}+t^{\frac{3}{2}}\right)-\left(\frac{2}{3}t^{\frac{3}{2}}+\frac{2}{5}t^{\frac{5}{2}}+5\right)}{(1+t)^2}$ Alt: $P+(1+t)\dfrac{\mathrm{d}P}{\mathrm{d}t}=t^{\frac{1}{2}}+t^{\frac{3}{2}}\Rightarrow\dfrac{\mathrm{d}P}{\mathrm{d}t}=\dfrac{t^{\frac{1}{2}}+t^{\frac{3}{2}}-\dfrac{\frac{2}{3}t^{\frac{3}{2}}+\frac{2}{5}t^{\frac{5}{2}}+5}{(1+t)}}{(1+t)}$	M1 A1ft	3.4 1.1b
	$\left(\dfrac{\mathrm{d}P}{\mathrm{d}t}\right)_{t=1}=\dfrac{\mathrm{d}P}{\mathrm{d}t}=\dfrac{5\times10-\left(\frac{16}{3}+\frac{64}{5}+5\right)}{(5)^2}=\dfrac{403}{375}$	dM1	3.1a
	$\dfrac{403}{375}\times1000=\dfrac{3224}{3}(=awrt\,1070)$ bacteria per hour	A1	3.2a
		(4)	
	(b) Alternative:		
	$P=\dfrac{\frac{2}{3}t^{\frac{3}{2}}+\frac{2}{5}t^{\frac{5}{2}}+5}{(1+t)}=\dfrac{\frac{16}{3}+\frac{64}{5}+5}{(1+4)}$	M1	3.4
	$=\dfrac{347}{75}$	A1ft	1.1b
	$(1+t)\dfrac{\mathrm{d}P}{\mathrm{d}t}+P=t^{\frac{1}{2}}(1+t)\Rightarrow5\dfrac{\mathrm{d}P}{\mathrm{d}t}+\dfrac{347}{75}=2\times5\Rightarrow\dfrac{\mathrm{d}P}{\mathrm{d}t}=\dfrac{403}{375}$	dM1	3.1a
	$\dfrac{403}{375}\times1000=\dfrac{3224}{3}(=1075)$ bacteria per hour	A1	3.2a
		(4)	
(c)	E.g. • The number of bacteria increases indefinitely which is not realistic	B1	3.5b
		(1)	
			(11 marks)

Notes
(a) B1: A correct rearrangement (may be implied by subsequent work). Alternatively, recognises the LHS as a derivative and writes $(1+t)\frac{\mathrm{d}P}{\mathrm{d}t}+P=\frac{\mathrm{d}}{\mathrm{d}t}(P(1+t))\left(=t^{\frac{1}{2}}(1+t)\right)$ (may be implied). M1: Uses the model to find the integrating factor (or recognise the derivative) and attempts the solution of the differential equation to achieve $P\times$ their IF $=\int$ their $t^{\frac{1}{2}}\times$ their IF $\mathrm{d}t=\ldots$ but do not be too concerned with the mechanics of integrating the RHS but it must be attempted. A1: Correct solution M1: Interprets the initial conditions to find the constant of integration. Must be using $t=0$ and $P=5$ in an equation with a constant of integration, but their equation may have come from incorrect work. This is correctly interpreting the initial conditions and attempting to use them. M1: Uses their solution to the problem to find the population after 8 hours. Must be using their solution, but allow for any equations which arise from an attempt at solving the differential equation. A1: **cso** Correct number of bacteria (accept awrt 10 300) from a correct equation (b) M1: Realises the need to differentiate the model and uses an appropriate method to find the derivative. Allow the M for attempts at implicit differentiation with (1+t)P = ... Trivialised differentiation from incorrect work is M0. A1ft: Correct differentiation of the correct answer to (a) up to the constant of integration to obtain dP/dt in terms of t (if implicit differentiation is used, they must get to a function in terms of t only, or revert to the Alternative method). Follow through on their c in an otherwise correct equation from (a). M1: Uses $t=4$ in their dP/dt (allow from any attempts at the derivative) to obtain a value for dP/dt. A1: Correct answer, allow 1075 or answers rounding down to 1070 with correct units. Accept as 1.07 thousand bacteria per hour. (NB If 5000 is used in (a) instead of 5, the answer here would be −198.725) **Alternative:** M1: Substitutes $t=4$ into their P A1ft: Correct value for P. Follow through on their constant of integration from part (a), but the rest of the equation must be correct. M1: Uses $t=4$ and their P to find a value for dP/dt A1: Correct answer allow 1075 or answers rounding down to 1070 with correct units. Accept as 1.07 thousand bacteria per hour. (c) B1: Suggests a suitable limitation which must refer to the model. Allow for a sensible comment even if they have no equation for the model Do not allow answers such as "the model does not take account of external factors such as temperature" as we do not know what factors the model does take account of.