

Question	Scheme	Marks	AOs
<b>2 (a)</b>	$\cos^2 \frac{x}{3} = \left( 1 - \frac{\left(\frac{x}{3}\right)^2}{2} + \frac{\left(\frac{x}{3}\right)^4}{24} - \dots \right)^2 \quad \text{or} \quad \left( 1 - \frac{x^2}{18} + \frac{x^4}{1944} - \dots \right)^2 = \dots \quad \text{or}$ $\frac{1}{2} \left( 1 \pm \cos \frac{2x}{3} \right) = \frac{1}{2} \left( 1 \pm \left( 1 - \frac{1}{2} \left( \frac{2x}{3} \right)^2 + \frac{1}{4!} \left( \frac{2x}{3} \right)^4 - \dots \right) \right)$	M1	2.2a
	$= 1 - \frac{x^2}{9} + \frac{1}{243}x^4$	A1	1.1b
	(2)		
<b>(b)</b>	$\int \frac{1 - \frac{x^2}{9} + \frac{1}{243}x^4}{x} = \int \frac{1}{x} - \frac{x}{9} + \frac{1}{243}x^3 = A \ln x + Bx^2 + Cx^4$ <p>where <math>A, B</math> and <math>C \neq 0</math></p>	M1	3.1a
	$\ln x - \frac{x^2}{18} + \frac{1}{972}x^4$	A1ft	1.1b
	= awrt 0.98295	A1	2.2a
	(3)		
<b>(c)</b>	Calculator = awrt 0.98280	B1	1.1b
		(1)	
<b>(d)</b>	E.g. the approximation is correct to 3 d.p.	B1	3.2b
		(1)	

**(7 marks)**

**Notes:**

**(a)**

**M1:** Deduces the required series by using the Maclaurin series for  $\cos x$ , replacing  $x$  with  $\frac{x}{3}$  and squares, or first applying the double angle identity (allow sign error) and then applying the series for  $\cos x$  with  $\frac{2x}{3}$ . Attempts at finding from differentiation score M0 as the cosine series is required.

**A1:** Correct series

**(b)**

**M1:** Divides their series in part (a) by  $x$  and integrates to the form  $A \ln x + Bx^2 + Cx^4$

**A1ft:** Correct integration, follow through on their coefficients and need not be simplified.

**A1:** Deduces the definite integral awrt 0.98295

**(c)**

**B1:** Correct value.

**(d)**

**B1:** Makes a quantitative statement about the accuracy, so e.g. how many decimal places or significant figures it is correct to, or calculates a percentage accuracy to deduce it is reasonable. Do not accept just “underestimate” or similar without quantitative evidence. Allow for a reasonable comment as long as (b) is correct to at least 2 s.f. but (c) must be the correct value.