Question	Scheme		Marks	AOs
8(a)	Volume of paint = 30 litres therefore Rate of paint out $=3\times\frac{r}{30}$ litres per second		M1	3.3
	$\frac{\mathrm{d}r}{\mathrm{d}t}=2-\frac{r}{10}$		A1	1.1b
			(2)	
(b)	Rearranges $\frac{\mathrm{d}r}{\mathrm{d}t}+\frac{r}{10}=2$ and attempts integrating factor IF = $\mathrm{e}^{\int\frac{1}{10}\mathrm{d}t}=\ldots$	Separates the variables $\int\frac{1}{20-r}\,\mathrm{d}r=\frac{1}{10}\,\mathrm{d}t$ $\Rightarrow\ldots$	M1	3.1a
	$r\mathrm{e}^{\frac{t}{10}}=\int 2\mathrm{e}^{\frac{t}{10}}\,\mathrm{d}t\Rightarrow r\mathrm{e}^{\frac{t}{10}}=\lambda\mathrm{e}^{\frac{t}{10}}(+c)$	Integrates to the form $\lambda\ln(20-r)=\frac{1}{10}t(+c)$	M1	1.1b
	$r\mathrm{e}^{\frac{t}{10}}=20\mathrm{e}^{\frac{t}{10}}+c$	$-\ln(20-r)=\frac{1}{10}t+c$	A1ft	1.1b
	$t=0,\ r=10\Rightarrow c=\ldots$		M1	3.4
	$r=\frac{20\mathrm{e}^{\frac{t}{10}}-10}{\mathrm{e}^{\frac{t}{10}}}=15$ rearranges to achieve $\mathrm{e}^{\frac{t}{10}}=\alpha$ and solves to find a value for t or $r=20-10\mathrm{e}^{-\frac{t}{10}}=15$ rearranges to achieve $\mathrm{e}^{-\frac{t}{10}}=\beta$ and solves to find a value for t	$-\ln(20-15)=\frac{1}{10}t-\ln10$ Leading to a value for t	M1	3.4
	$t=$ awrt 7 seconds		A1	2.2b
			(6)	
(c)	The model predicts 7 seconds but it actually takes 9 seconds so (over) 2 seconds out (over 20%), therefore it is not a good model		B1ft	3.5a
			(1)	
				(9 marks)

Notes:

(a)

M1: Clearly identifies that Rate of paint out $=3\times\frac{r}{\text{their volume}}$. It is a "show that" question so there must be clearly reasoning. Just answer with no reasoning scores M0.

A1: Puts all the components together to form the correct differential equation.

(b)
M1: Identifies as a first order differential equation and finds the integrating factor or separates the variables and integrates. Allow if there are sign slips in rearranging (e.g. to $\frac{\mathrm{d}r}{\mathrm{d}t}-\frac{r}{10}=2$) or in the integrating factor and allow with their value for *a* or with *a* as an unknown.
M1: Multiplies through by the IF and attempts to integrate or integrates to the form
$\lambda \ln(2a-r)=\frac{1}{a}t+c$ oe
A1ft: Correct integration, including constant of integration. Follow through on their value of *a*, but not sign slips from rearrangement. So allow for $re^{\frac{t}{a}}=2a\,e^{\frac{t}{a}}+c$ or $-\ln(2a-r)=\frac{1}{a}t+c$ oe with *a* or their *a*.
M1: Uses the initial conditions to find the constant of integration. Must see substitution or can be implied by the correct value for their equation. Allow for finding in terms of *a* if separation of variables used.

M1: Sets $r=15$, achieves $e^{\frac{t}{10}}=\alpha>0$ or $e^{-\frac{t}{10}}=\beta>0$ as appropriate and solves to find a value for *t*. Separates the variable method sets *r* = 15 and rearranges to find a value for *t*. **Note:** For this mark a value of *a* is needed, but need not be the correct one.
A1cso: $t=$ awrt 7 seconds from fully correct work.

(c)
B1ft: See scheme, follow through on their answer to part (b). Accept any reasonable comparative comment but must have a reason, not just a statement of good or not good. So e.g. look for finding the difference between their answer and 9, or the percentage difference. If their answer is close to 9, then accept a conclusion of being a good model if a suitable reason is given. May substitute 9 into their equation and obtain a value to compare with 15 and make a similar conclusion.