Question	Scheme	Marks	AOs
9(a)	$\int \frac{x^2}{\sqrt{x^2-1}}\,dx \rightarrow \int f(u)\,du$ Uses the substitution $x = \cosh u$ fully to achieve an integral in terms of u only, including replacing the dx	M1	3.1a
	$\int \frac{\cosh^2 u}{\sqrt{\cosh^2 u - 1}} \sinh u\,(du)$	A1	1.1b
	Uses correct identities $\cosh^2 u - 1 = \sinh^2 u$ and $\cosh 2u = 2\cosh^2 u - 1$ to achieve an integral of the form $A\int(\cosh 2u \pm 1)\,du \qquad A > 0$	M1	3.1a
	Integrates to achieve $A\left(\pm\frac{1}{2}\sinh 2u \pm u\right)(+c) \qquad A > 0$	M1	1.1b
	Uses the identity $\sinh 2u = 2\sinh u \cosh u$ and $\cosh^2 u - 1 = \sinh^2 u$ $\rightarrow \sinh 2u = 2x\sqrt{x^2-1}$	M1	2.1
	$\frac{1}{2}\left[x\sqrt{x^2-1} + \operatorname{ar}\cosh x\right] + k$ * cso	A1*	1.1b
		(6)	
(b)	Uses integration by parts the correct way around to achieve $\int \frac{4}{15} x \operatorname{arcosh} x\,dx = Px^2 \operatorname{arcosh} x - Q\int \frac{x^2}{\sqrt{x^2-1}}\,dx$	M1	2.1
	$= \frac{4}{15}\left(\underline{\frac{1}{2}x^2 \operatorname{arcosh} x - \frac{1}{2}\int \frac{x^2}{\sqrt{x^2-1}}\,dx}\right)$	A1	1.1b
	$= \frac{4}{15}\left(\frac{1}{2}x^2 \operatorname{arcosh} x - \frac{1}{2}\underline{\left(\frac{1}{2}\left[x\sqrt{x^2-1} + \operatorname{arcosh} x\right]\right)}\right)$	B1ft	2.2a
	Uses the limits $x = 1$ and $x = 3$ the correct way around and subtracts $= \frac{4}{15}\left(\frac{1}{2}(3)^2 \operatorname{arcosh} 3 - \frac{1}{2}\left(\frac{1}{2}\left[3\sqrt{(3)^2-1} + \operatorname{arcosh} 3\right]\right)\right) - \frac{4}{15}(0)$	dM1	1.1b
	$= \frac{4}{15}\left(\frac{9}{2}\ln\left(3+\sqrt{8}\right) - \frac{3\sqrt{8}}{4} - \frac{1}{4}\ln\left(3+\sqrt{8}\right)\right)$ $= \frac{1}{15}\left[17\ln\left(3+2\sqrt{2}\right) - 6\sqrt{2}\right]$ *	A1*	1.1b
		(5)	
			(11 marks)

Notes:

(a)

M1: Uses the substitution $x = \cosh u$ fully to achieve an integral in terms of *u* only. Must have replaced the d*x* but allow if the d*u* is missing.

A1: Correct integral in terms of *u*. (Allow if the d*u* is missing.)

M1: Uses correct identities $\cosh^2 u - 1 = \sinh^2 u$ and $\cosh 2u = 2\cosh^2 u - 1$ to achieve an integrand of the required form

M1: Integrates to achieve the correct form, may be sign errors.

M1: Uses the identities $\sinh 2u = 2\sinh u \cosh u$ and $\cosh^2 u - 1 = \sinh^2 u$ to attempt to find $\sinh 2u$ in terms of *x*. If using exponentials there must be a full and complete method to attempt the correct form.

A1*: Achieves the printed answer with no errors seen, cso

NB attempts at integration by parts are not likely to make progress – to do so would need to split the integrand as $x\dfrac{x}{\sqrt{x^2-1}}$. If you see any attempts that you feel merit credit, use review.

(b)

M1: Uses integration by parts the correct way around to achieve the required form.

A1: Correct integration by parts

B1ft: Deduces the integral by using the result from part (a). Follow through on their '*uv*'

dM1: Dependent on previous method mark. Uses the limits $x = 1$ and $x = 3$ the correct way around and subtracts

A1*cso: Achieves the printed answer with at least one intermediate step showing the evaluation of the arcosh 3, and no errors seen.