

Question	Scheme	Marks	AOs
2(a)	$z = \cos \theta + i \sin \theta \Rightarrow \frac{1}{z} = \cos \theta - i \sin \theta$ $\Rightarrow \left(z + \frac{1}{z}\right)^5 = (2 \cos \theta)^5 = 32 \cos^5 \theta$	M1	2.1
	$\left(z + \frac{1}{z}\right)^5 = z^5 + \frac{1}{z^5} + 5\left(z^3 + \frac{1}{z^3}\right) + 10\left(z + \frac{1}{z}\right)$	M1 A1	2.1 1.1b
	$= 2 \cos 5\theta + 10 \cos 3\theta + 20 \cos \theta$	M1	2.1
	$\cos^5 \theta = \frac{1}{16} (\cos 5\theta + 5 \cos 3\theta + 10 \cos \theta)^*$	A1*	1.1b
		(5)	
(b)	$\cos \theta - \cos 5\theta = 5 \cos 3\theta \Rightarrow \cos \theta = 5 \cos 3\theta + \cos 5\theta = 16 \cos^5 \theta - 10 \cos \theta$	M1	3.1a
	$16 \cos^5 \theta - 11 \cos \theta = 0$	A1	1.1b
	$\cos \theta (16 \cos^4 \theta - 11) = 0 \Rightarrow \cos \theta = 0, \pm \sqrt[4]{\frac{11}{16}}$	M1	1.1b
	$\theta = 3.57, \frac{3\pi}{2} \text{ (or } 4.71), 5.86$	A1 A1	1.1b 1.1b
		(5)	

(10 marks)

Notes

(a)

M1: Begins the proof by demonstrating that $\left(z + \frac{1}{z}\right)^5 = 32 \cos^5 \theta$

M1: Attempts to expand $\left(z + \frac{1}{z}\right)^5$ including the binomial coefficients

A1: Correct expansion

M1: Uses $z^n + \frac{1}{z^n} = 2 \cos n\theta$ to obtain an expression in terms of $\cos 5\theta$, $\cos 3\theta$ and $\cos \theta$

A1*: Concludes the argument by equating the two expressions leading to the printed answer with no errors

(b)

M1: Makes the connection with part (a) and reaches an equation in $\cos \theta$ only

A1: Correct equation

M1: Solves their equation for $\cos \theta$

A1: 2 correct solutions

A1: All 3 correct solutions. Ignore extra solutions outside the range but deduct this mark if there are extra answers in range.