Question	Scheme	Marks	AOs
8(a)	$y=\frac{\mathrm{d}x}{\mathrm{d}t}+5x-51 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+5\frac{\mathrm{d}x}{\mathrm{d}t}$	B1	2.1
	$\Rightarrow \frac{\mathrm{d}^2x}{\mathrm{d}t^2}+5\frac{\mathrm{d}x}{\mathrm{d}t}=12x-6\left(\frac{\mathrm{d}x}{\mathrm{d}t}+5x-51\right)$	M1	2.1
	$\Rightarrow \frac{\mathrm{d}^2x}{\mathrm{d}t^2}+11\frac{\mathrm{d}x}{\mathrm{d}t}+18x=306$ *	A1*	1.1b
		(3)	
(b)	$m^2+11m+18=0 \Rightarrow m=...$	M1	3.4
	$m=-2,-9$	A1	1.1b
	$x=A\mathrm{e}^{\alpha t}+B\mathrm{e}^{\beta t}$	M1	3.4
	$x=A\mathrm{e}^{-9t}+B\mathrm{e}^{-2t}$	A1	1.1b
	PI: Try $x=k \Rightarrow 18k=306$ $\Rightarrow k=17$	M1	3.4
	$GS: x=A\mathrm{e}^{-9t}+B\mathrm{e}^{-2t}+17$	A1ft	1.1b
		(6)	
(c)	$y=\frac{\mathrm{d}x}{\mathrm{d}t}+5x-51 \Rightarrow y=-9A\mathrm{e}^{-9t}-2B\mathrm{e}^{-2t}+5A\mathrm{e}^{-9t}+5B\mathrm{e}^{-2t}+85-51$	M1	3.4
	$y=3B\mathrm{e}^{-2t}-4A\mathrm{e}^{-9t}+34$	A1	1.1b
		(2)	
(d)	$0=A+B+17,\ 0=3B-4A+34 \Rightarrow A=...,B=...$ (NB $A=-\frac{17}{7}, B=-\frac{102}{7}$)	M1	3.3
	$x=17-\frac{17}{7}\mathrm{e}^{-9t}-\frac{102}{7}\mathrm{e}^{-2t}, y=34+\frac{68}{7}\mathrm{e}^{-9t}-\frac{306}{7}\mathrm{e}^{-2t}$	A1	1.1b
	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{\mathrm{d}y}{\mathrm{d}t} \Rightarrow \frac{153}{7}\mathrm{e}^{-9t}+\frac{204}{7}\mathrm{e}^{-2t}=-\frac{612}{7}\mathrm{e}^{-9t}+\frac{612}{7}\mathrm{e}^{-2t} \Rightarrow \mathrm{e}^{k}=\alpha$	M1	3.1b
	$\mathrm{e}^{7t}=\frac{15}{8} \Rightarrow 7t=\ln\left(\frac{15}{8}\right) \Rightarrow t=\frac{1}{7}\ln\left(\frac{15}{8}\right)$	M1	1.1b
	= 5.39 minutes	A1	3.2a
		(5)	
(e)	E.g. • The model suggests that, in the long term, the amount of antibiotic in the blood (and/or the body tissue) will remain constant and this is unlikely	B1	3.5a
		(1)	
			(17 marks)

Notes

(a)

B1: Differentiates the first equation with respect to t correctly

M1: Proceeds to the printed answer by substituting into the second equation

A1*: Achieves the printed answer with no errors

(b)
M1: Uses the model to form and solve the Auxiliary Equation
A1: Correct roots of the AE
M1: Uses the model to form the Complementary Function
A1: Correct CF
M1: Chooses the correct form of the PI according to the model and uses a complete method to find the PI
A1ft: Combines their CF and PI to give x in terms of t
(c)
M1: Uses the model and their answer to part (b) to give y in terms of t
A1: Correct equation
(d)
M1: Realises the need to use the initial conditions to establish the values of their constants
A1: Correct particular solutions for x and y
M1: Differentiates both expressions, sets them equal and proceeds to reach an equation of the form $e^k = \alpha$
M1: Correct use of logarithms to reach $t = \ldots$
A1: Correct value
(e)
B1: Suggests a suitable evaluation of the model