

Figure 1

Figure 1 shows the design for a new type of security wheel nut for a car. The inner circle has a radius of 5 mm and the outer circle has a radius of 10 mm. The curve, C, between the two circles, is modelled by the polar equation

$$r = 7.5 + 1.5\cos 6\theta$$
 $0 \le \theta < 2\pi$

where r is measured in millimetres.

The regions R_1 and R_2 are shown shaded in Figure 1 and both regions must be coated in a special paint.

The region R_1 is enclosed between the outer circle and C.

The region R_2 is enclosed by the inner circle.

Find the area that must be coated in the special paint, according to the model.

Give your answer in cm² to 2 decimal places.

[Solutions based entirely on graphical or numerical methods are not acceptable.]