**8.** A doctor is studying the concentration of an antibiotic in the blood and the body tissue of a patient.
Let $x$ be the number of micrograms of the antibiotic in the blood.
Let $y$ be the number of micrograms of the antibiotic in the body tissue.
The doctor models her results by the differential equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -5x + y + 51$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 12x - 6y$$

where $t$ is the time in hours after a dose of the antibiotic has been administered to the patient.

(a) Show that

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 11\frac{\mathrm{d}x}{\mathrm{d}t} + 18x = 306$$

**(3)**

(b) Find a general solution for the number of micrograms of the antibiotic in the blood at time $t$ hours.

**(6)**

(c) Hence find a general solution for the number of micrograms of the antibiotic in the body tissue at time $t$ hours.

**(2)**

Initially there is none of this antibiotic in the blood and none of this antibiotic in the body tissue.

(d) Find, in minutes, to 2 decimal places, the time when the rate of increase of the antibiotic in the blood is equal to the rate of increase of the antibiotic in the body tissue.

**(5)**

(e) Evaluate the model.

**(1)**