

Question	Scheme	Marks	AOs
2	Solves the quadratic equation for $\cosh^2 x$ e.g. $(8 \cosh^2 x - 9)(8 \cosh^2 + 1) = 0 \Rightarrow \cosh^2 x = \dots$	M1	3.1a
	$\cosh^2 x = \frac{9}{8} \left\{ -\frac{1}{8} \right\}$	A1	1.1b
	$\cosh x = \frac{3}{4}\sqrt{2} \Rightarrow x = \ln \left[ \frac{3}{4}\sqrt{2} + \sqrt{\left(\frac{3}{4}\sqrt{2}\right)^2 - 1} \right]$ <b>Alternatively</b> $\cosh x = \frac{3}{4}\sqrt{2} \Rightarrow \frac{1}{2}(e^x + e^{-x}) \Rightarrow e^{2x} - \frac{3}{2}\sqrt{2}e^x + 1 = 0$ $\Rightarrow e^x = \sqrt{2}$ or $\frac{\sqrt{2}}{2} \Rightarrow x = \dots$	M1	1.1b
	$x = \pm \frac{1}{2} \ln 2$	A1	2.2a
		(4)	

(4 marks)

**Notes:**

**M1:** Solves the quadratic equation for  $\cosh^2 x$  by any valid means. If by calculator accept for reaching the positive value for  $\cosh^2 x$  (negative may be omitted or incorrect) but do not allow for going directly to a value for  $\cosh x$ . Alternatively score a correct process leading to a value for  $\sinh 2x$  or its square (Alt 1) or use of correct exponential form for  $\cosh x$  to form and expand to an equation in  $e^{4x}$  and  $e^{2x}$  (Alt 2)

**A1:** Correct value for  $\cosh^2 x$  (ignore negative or incorrect extra roots.). In Alt 1 score for a correct value for  $\sinh^2 2x$  or  $\sinh 2x$ . In Alt 2 score for a correct simplified equation in  $e^{4x}$ .

**M1:** For a correct method to achieve at least one value for  $x$  (from  $\cosh^2 x$ ). In the main scheme or Alt 1, takes positive square root (if appropriate) and uses the correct formula for  $\operatorname{arcosh} x$  or  $\operatorname{arsinh} x$  to find a value for  $x$ . (No need to see negative square root rejected.) In Alt 2 it is for solving the quadratic in  $e^{4x}$  and proceeding to find a value for  $x$ .

**Alternatively** uses the exponential definition for  $\cosh x$ , forms and solves a quadratic for  $e^x$  leading to a value for  $x$

**A1:** Deduces (both) the correct values for  $x$  and no others. Must be in the form specified.

SC Allow M0A0M1A1 for cases where a calculator was used to get the value for  $\cosh x$  with no evidence if a correct method for find both values is shown.

**2 Alt 1**

$$64 \cosh^2 x (\cosh^2 x - 1) - 9 = 0 \Rightarrow 64 \cosh^2 x \sinh^2 x - 9 = 0$$

$$\Rightarrow 16 \sinh^2 2x = 9 \Rightarrow \sinh^2 2x = \frac{9}{16}$$

$$\text{Or } (8 \sinh x \cosh x - 3)(8 \sinh x \cosh x + 3) = 0 \Rightarrow \sinh 2x = \pm \frac{3}{4}$$

M1

3.1a

A1

1.1b

$$\sinh 2x = \pm \frac{3}{4} \Rightarrow x = \frac{1}{2} \ln \left[ \pm \frac{3}{4} + \sqrt{\frac{9}{16} + 1} \right] \text{ (or use exponentials, or proceed via cosh4x)}$$

M1

1.1b

$$x = \pm \frac{1}{2} \ln 2$$

A1

2.2a

**(4)****2 Alt 2**

$$64 \left( \frac{e^x + e^{-x}}{2} \right)^4 - 64 \left( \frac{e^x + e^{-x}}{2} \right)^2 - 9 = 0 \Rightarrow$$

$$4(e^{4x} + 4e^{2x} + 6 + 4e^{-2x} + e^{-4x}) - 16(e^{2x} + 2 + e^{-2x}) - 9 = 0$$

$$4e^{4x} - 17 + 4e^{-4x} = 0$$

M1

3.1a

A1

1.1b

$$(4e^{4x} - 1)(1 - 4e^{-4x}) = 0 \Rightarrow e^{4x} = \dots \Rightarrow x = \dots$$

M1

1.1b

$$x = \pm \frac{1}{2} \ln 2$$

A1

2.2a

**(4)**