Question	Scheme	Marks	AOs
4(a)	Applies $ln\left(\frac{r+1}{r-1}\right) = ln(r + 1) - ln(r - 1)$ to the problem in order to apply differences.	M1	3.1a
	$$\sum_{r=2}^{n}(ln(r + 1) - ln(r - 1))$$ $$= (ln(3) - ln(1)) + (ln(4) - ln(2)) + (ln(5) - ln(3))+\ldots$$ $$+(ln(n) - ln(n - 2)) + (ln(n + 1) - ln(n - 1))$$	dM1	1.1b
	$ln(n) + ln(n + 1) - ln\, 2$	A1	1.1b
	$ln\left(\frac{n(n+1)}{2}\right)$ * cso	A1 *	2.1
		(4)	
(b)	$$\sum_{r=51}^{100} ln\left(\frac{r+1}{r-1}\right) = \sum_{r=2}^{100} ln\left(\frac{r+1}{r-1}\right) - \sum_{r=2}^{50} ln\left(\frac{r+1}{r-1}\right)$$ $$= ln\left(\frac{100 \times 101}{2}\right) - ln\left(\frac{50 \times 51}{2}\right)$$	M1	1.1b
	$\sum_{r=51}^{100} ln\left(\frac{r+1}{r-1}\right)^{35} = 35\, ln\left(\frac{100\times101}{2} \div \frac{50\times51}{2}\right)$	M1	3.1a
	$= 35\, ln\left(\frac{202}{51}\right)$	A1	1.1b
		(3)	
		(7 marks)	

Notes:

(a)

M1: Uses the subtraction laws of logs to start the method of differences process.

dM1: Demonstrates the method of differences process, should have a minimum of e.g $r = 2, r = 3, r = 4,\ r = n - 1$ and $r = n$ shown -- enough to establish *at least one cancelling term* and *all non-disappearing terms* though the latter may be implied by correct extraction if only the first few cases are shown. Allow this mark if an extra term for $r = 1$ has been included.

A1: Correct terms that do not cancel - must not contradict their list of terms so e.g. if $r = 1$ was included, then A0A0 follows. The ln 1 may be included for this mark.

A1*: Achieves the printed answer, with no errors or omissions **and** must have had a complete list (as per dM1) before extraction (but condone missing brackets on ln terms). If working with r throughout, they must replace by n to gain the last A, but all other marks are available.

NB For attempts at combining log terms instead of using differences, full marks may be awarded for the equivalent steps, but attempts that do not make progress in combining terms will score no marks.

(b) Condone a bottom limit of 0 or 1 being used throughout part (b).
M1: Attempts to split into (the sum up to 100) – (the sum up to k) where k is 49, 50 or 51 **and** apply the result of (a) in some way. Condone slips with the power.

M1: Having attempted to apply (a), uses difference and power log laws correctly to reach an expression of the required form.

A1: Correct answer. Accept equivalents in required form, such as $35\, ln\frac{5050}{1275}$