

Question	Scheme	Marks	AOs
9(i) (a)	E. g. <ul style="list-style-type: none"> ● Because the interval being integrated over is unbounded. ● cosh x is undefined at the limit of ∞ ● the upper limit is infinite 	B1	1.2
		(1)	
(i) (b)	$\int_0^{\infty} \cosh x \, dx = \lim_{t \rightarrow \infty} \int_0^t \cosh x \, dx$ or $\lim_{t \rightarrow \infty} \int_0^t \frac{1}{2}(e^x + e^{-x}) \, dx$	B1	2.5
	$\int_0^t \cosh x \, dx = [\sinh x]_0^t = \sinh t (-0)$ or $\frac{1}{2} \int_0^t e^x + e^{-x} \, dx = \frac{1}{2}[e^x - e^{-x}]_0^t = \frac{1}{2}[e^t - e^{-t}] \left[-\frac{1}{2}[e^0 - e^0] \right]$	M1	1.1b
	When $t \rightarrow \infty$ $e^t \rightarrow \infty$ and $e^{-t} \rightarrow 0$ therefore the integral is divergent	A1	2.4
		(3)	
(ii)	$4 \sinh x = p \cosh x \Rightarrow \tanh x = \frac{p}{4}$ or $4 \tanh x = p$ Alternative $\frac{4}{2}(e^x - e^{-x}) = \frac{p}{2}(e^x + e^{-x}) \Rightarrow 4e^x - 4e^{-x} = pe^x + pe^{-x}$ $e^{2x}(4 - p) = p + 4 \Rightarrow e^{2x} = \frac{p + 4}{4 - p}$	M1	3.1a
	$\left\{ -1 < \frac{p}{4} < 1 \Rightarrow \right\} -4 < p < 4$	A1	2.2a
		(2)	

(6 marks)

(i)(a)

B1: For a suitable explanation. Technically this should refer to the interval being unbounded, but this is unlikely to be seen. Accept “Because the upper limit is infinity”, but **not** “because it is infinity” without reference to what “it” is. Do not accept “the upper limit tends to infinity” or “the integral is unbounded”.

(i)(b)

B1: Writes the integral in terms of a limit as $t \rightarrow \infty$ (or other variable) with limits 0 and “ t ”, or implies the integral is a limit by subsequent working by correct language.

M1: Integrates cosh x correctly either as $\sinh x$ or in terms of exponentials and applies correctly the limits of 0 and “ t ”. The bottom limit zero may be implied. No need for the $\lim_{t \rightarrow \infty}$ for this mark but substitution of ∞ is M0.

A1: cso States that (as $t \rightarrow \infty$) $\sinh t \rightarrow \infty$ or $e^t \rightarrow \infty$ and $e^{-t} \rightarrow 0$ therefore divergent (or not convergent), or equivalent working. Accept $\sinh t$ is undefined as $t \rightarrow \infty$

(ii)

M1: Divides through by cosh x to find an expression involving tanh x

Alternative: uses the correct exponential definitions and finds an expression for e^{2x} or solves a quadratic in e^{2x}

A1: Deduces the correct inequality for p . Note $|p| < 4$ is a correct inequality for p .