

Question	Scheme	Marks	AOs
2(a)	$x^2 + 4x - 5 = (x+2)^2 - 9$	B1	1.1b
		(1)	
(b)	$\int \frac{1}{\sqrt{(x+p)^2 - q}} dx = \operatorname{arcosh}\left(\frac{x+p}{\sqrt{q}}\right) (+c) \quad \text{or}$ $\ln\left(x+p + \sqrt{(x+p)^2 - q}\right) (+c)$	M1	1.1a
	$= \operatorname{arcosh}\left(\frac{x+2}{3}\right) \quad \text{or} \quad \ln\left(x+2 + \sqrt{(x+2)^2 - 9}\right) \quad \text{oe}$	A1	2.2a
		(2)	
(c)	$\text{Mean} = \frac{1}{13-3} \int_3^{13} \frac{1}{\sqrt{x^2 + 4x - 5}} dx$	B1	1.2
	$\frac{1}{10} \int_3^{13} \frac{1}{\sqrt{x^2 + 4x - 5}} dx = \frac{1}{10} \left(\operatorname{arcosh}\left(\frac{15}{3}\right) - \operatorname{arcosh}\left(\frac{5}{3}\right) \right)$ or	M1	1.1b
	$\frac{1}{10} \int_3^{13} \frac{1}{\sqrt{x^2 + 4x - 5}} dx = \frac{1}{10} \left(\ln(15 + \sqrt{216}) - \ln(5 + \sqrt{16}) \right)$		
	$= \frac{1}{10} \ln\left(\frac{5+2\sqrt{6}}{3}\right) \quad \text{or} \quad \frac{1}{20} \ln\left(\frac{49+20\sqrt{6}}{9}\right)$	A1	3.2a
		(3)	

(6 marks)

Notes:

(a)
B1: Correct completed square form. Allow 3^2 for 9.

(b)
M1: Achieves a correct form for the integration for their p and q from part (a):

$$\operatorname{arcosh}\left(\frac{x+p}{\sqrt{q}}\right) (+c) \quad \text{or} \quad \ln\left(x+p + \sqrt{(x+p)^2 - q}\right) (+c) \quad \text{or e.g.} \quad \ln\left(\frac{x+p}{\sqrt{q}} + \sqrt{\left(\frac{x+p}{\sqrt{q}}\right)^2 - 1}\right) (+c)$$

where $p \neq 0, q \neq 1$

Allow \cosh^{-1} for arcosh

Allow attempts that use substitution following an attempt to complete the square but must be an appropriate substitution e.g. $x+p = \sqrt{q} \cosh u$ leading to a correct form as above.

A1: Correct integration. The “+ c” is not required. Apply isw once a correct expression is seen.

Note that $\ln\left(\frac{x+2}{3} + \sqrt{\left(\frac{x+2}{3}\right)^2 - 1}\right) (+c)$ is also correct

(c)

B1: Recalls the definition of a mean function accurately. $\frac{1}{13-3} \int_3^{13} \frac{1}{\sqrt{x^2 + 4x - 5}} dx$ seen or implied.

Note that the $\frac{1}{13-3}$ may appear at the end. $\frac{1}{13-3} \int_3^{13} f(x) dx$ is sufficient as $f(x)$ is defined in the question. Also allow it to be implied by e.g. $\frac{1}{10} [g(x)]_3^{13}$ where $g(x)$ is their integrated function.

M1: Applies the correct limits the right way round to whatever they think the answer to part (b) is.

This can be awarded if the $\frac{1}{10}$ is present or not.

A1: Correct answer in correct form. Allow equivalents e.g. $\frac{1}{10} \ln \left(\frac{5}{3} + \frac{2\sqrt{6}}{3} \right)$, $\frac{1}{20} \ln \left(\frac{49}{9} + \frac{20\sqrt{6}}{9} \right)$

And allow if the surd is not simplified e.g. $\frac{1}{10} \ln \left(\frac{5 + \sqrt{24}}{3} \right)$, $\frac{1}{20} \ln \left(\frac{49 + \sqrt{2400}}{9} \right)$

Apply isw once a correct answer is seen.

The brackets must be present in forms such as $\frac{1}{10} \ln \left(\frac{5}{3} + \frac{2\sqrt{6}}{3} \right)$, $\frac{1}{20} \ln \left(\frac{49}{9} + \frac{20\sqrt{6}}{9} \right)$ but not in

e.g. $\frac{1}{10} \ln \frac{5 + \sqrt{24}}{3}$

If extra values are offered then score A0