Question	Scheme	Marks	AOs
4	$\begin{pmatrix}1 & -2\\0 & 1\end{pmatrix}^n = \begin{pmatrix}1 & -2n\\0 & 1\end{pmatrix}$		
	$\begin{pmatrix}1 & -2\\0 & 1\end{pmatrix}^1 = \begin{pmatrix}1 & -2\times1\\0 & 1\end{pmatrix}$ (so true when $n = 1$)	**B1**	2.2a
	(Assume true for $n = k$, then) $\begin{pmatrix}1 & -2\\0 & 1\end{pmatrix}^{k+1} = \begin{pmatrix}1 & -2\\0 & 1\end{pmatrix}\begin{pmatrix}1 & -2k\\0 & 1\end{pmatrix}$ or $\begin{pmatrix}1 & -2k\\0 & 1\end{pmatrix}\begin{pmatrix}1 & -2\\0 & 1\end{pmatrix}$	**M1**	2.1
	$= \begin{pmatrix}1 & -2k-2\\0 & 1\end{pmatrix}$ or $= \begin{pmatrix}1 & -2-2k\\0 & 1\end{pmatrix}$	**A1**	1.1b
	$= \begin{pmatrix}1 & -2(k+1)\\0 & 1\end{pmatrix}$	**A1**	2.2a
	Hence result is true for $n = k+1$. As true for $n = 1$ and have shown if true for $n = k$ then it is true for $n = k+1$, so it is true for all n.	**A1**	2.5
		(5)	
			(5 marks)

Notes:

B1: Shows true for $n = 1$.

Need to see $n = 1$ **substituted** into rhs. The minimum for B1 would be $\begin{pmatrix}1 & -2\times1\\0 & 1\end{pmatrix} = \begin{pmatrix}1 & -2\\0 & 1\end{pmatrix}$.

There is no need to state "True for $n = 1$"

M1: (Assumes for $n = k$ and) multiplies original matrix by kth power matrix either way round. Note that the assumption statement is not needed for this mark (but see below) so just look for:

$$\begin{pmatrix}1 & -2\\0 & 1\end{pmatrix}\begin{pmatrix}1 & -2k\\0 & 1\end{pmatrix} \text{ or } \begin{pmatrix}1 & -2k\\0 & 1\end{pmatrix}\begin{pmatrix}1 & -2\\0 & 1\end{pmatrix}$$

A1: Achieves a correct unsimplified matrix.

A1: Reaches correct form for the matrix with the $k + 1$ factored out with no errors and the correct unsimplified matrix seen previously. Note that the result may be proved by equivalence (see below).

A1: Correct conclusion **with assumption made** (which may be implied in their conclusion if they say "if true for $n = k$ then…"). This mark is dependent on all the previous marks apart from the B mark and is gained by conveying all the underlined points.

Allow this mark to score as long as all the underlined points are seen as narrative in their solution.

There must be the assumption statement somewhere or the "if…then…" idea in the conclusion. If awarded for the assumption statement condone e.g. true for $n = k$ in the conclusion.

The conclusion must convey the "if true for $n = k$ then true for $n = k + 1$" idea and not e.g. true for k, true for $k + 1$, true for 1 therefore…but see the previous note.

For the "true for all n" part condone e.g. "true for n", "true for all integers after 1", "true for $\mathbb{Z}^+$",
But do **not** allow "true for all values", "true for all real numbers"

Q4 Extra Notes:

1. For candidates who use *n* instead of *k* throughout withhold the final mark if the work is otherwise correct.

2. For equivalence proofs, this would be minimally acceptable:

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2\times 1 \\ 0 & 1 \end{pmatrix} \quad \textbf{B1}$$

"Need to prove" oe e.g. "target is" , "$n = k + 1$", "need" etc. $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & -2k-2 \\ 0 & 1 \end{pmatrix}$

Note that there must be a reference to $k + 1$ as above or e.g. (Target =) $\begin{pmatrix} 1 & -2(k+1) \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2k-2 \\ 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2k \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2k-2 \\ 0 & 1 \end{pmatrix} \quad \textbf{M1A1A1}$$

Hence result is true for $n = k + 1$. As <u>true for $n = 1$</u> and have shown <u>if true for $n = k$ then it is true for $n = k + 1$</u>, so it is <u>true for all n</u>. **A1**

Without the "Need to prove…" oe the response would score B1M1A1 and then A1 if the factorised form was shown or equivalence shown and then A1 for the correct conclusion.

3. Allow e.g. "correct" for "true" in the conclusion