Question	Scheme	Marks	AOs
5(a)	$2(\lambda-5)+3(-3\lambda-4)-2(5\lambda+3)=6 \Rightarrow \lambda=...(-2)$ $\lambda="-2" \Rightarrow x=...$ or $y=...$ or $z=...$ or e.g. $2x+3(-3x-15-4)-2(5x+25+3)=6 \Rightarrow x=...$	M1	1.1b
	$(-7,\ 2,-7)$	A1	1.1b
		(2)	
(b)	E.g. $\mathbf{r}=\begin{pmatrix}-5\\-4\\3\end{pmatrix}+\begin{pmatrix}2t\\3t\\-2t\end{pmatrix}$ meets the plane when $2(-5+2t)+3(-4+3t)-2(3-2t)=6 \Rightarrow t=...$	M1	3.1a
	$t=2 \Rightarrow$ mirror point is $\begin{pmatrix}-5\\-4\\3\end{pmatrix}+\begin{pmatrix}2\times4\\3\times4\\-2\times4\end{pmatrix}=...$	M1	1.1b
	$=\begin{pmatrix}3\\8\\-5\end{pmatrix}$	A1	1.1b
	$\mathbf{r}=\begin{pmatrix}-7\\2\\-7\end{pmatrix}+\mu\begin{pmatrix}3-(-7)\\8-2\\-5-(-7)\end{pmatrix}=...$	ddM1	1.1b
	$\mathbf{r}=\begin{pmatrix}-7\\2\\-7\end{pmatrix}+\mu\begin{pmatrix}10\\6\\2\end{pmatrix}$ *	A1*	2.1
		(5)	
	(b) Alternative for first 2 marks:		
	Distance from (–5, –4, 3) to plane is $\dfrac{\lvert 2\times-5+3\times-4-2\times3-6\rvert}{\sqrt{2^2+3^2+2^2}}=2\sqrt{17}$	M1	3.1a
	$\left\lvert\begin{matrix}2k\\3k\\-2k\end{matrix}\right\rvert=4\sqrt{17} \Rightarrow 4k^2+9k^2+4k^2=16\times17 \Rightarrow k=4$ $k=4 \Rightarrow$ mirror point is $\begin{pmatrix}-5\\-4\\3\end{pmatrix}+\begin{pmatrix}2\times4\\3\times4\\-2\times4\end{pmatrix}=...$	M1	1.1b

Do not penalise the omission of "r = " more than once in this question so penalise only once and on its first occurrence.

Notes:

(a) **Correct answer only scores no marks.**

M1: Substitutes the parametric form of the line into the plane and solves for their parameter and uses this to find at least one coordinate. Alternatively substitutes line equation into the plane equation to obtain and solve an equation in one variable.

A1: Correct point. Accept as $x=-7,\ y=2,\ z=-7$ or as a vector.

(b)

M1: Identifies a point on l_1 and uses the equation of the line through this point perpendicular to the plane to find the parameter where it intersects the plane. May use a different starting point.

M1: Uses twice their parameter to find the image point of their starting point in the plane.

A1: Correct image point.

ddM1: Finds the equation of the line between their intersection point from (a) and their image point. May be implied if they use e.g. 3**i** + 8**j** – 5**k** as the position vector. Alternatively, checks that the image point satisfies the equation for the other line. **Depends on both previous method marks.**

A1*: Fully correct work with conclusion that hence the given equation gives the required line. Must be as printed with "**r** = " but condone use of a different parameter e.g. t, λ etc. If they checked that the image point satisfied the equation of line 2 they would need to then say e.g. that both lines pass through (–7, 2, –7) and make a minimal conclusion.

Alternative for first 2 marks in (b):

M1: Identifies a point on l_1 and finds the perpendicular distance from this point to the plane. May use a different starting point.

M1: Uses twice their distance to find the image point of their starting point in the plane.

(c)	Line joining mirror points intersects plane at $\begin{pmatrix}-5\\-4\\3\end{pmatrix}+\begin{pmatrix}2\times2\\3\times2\\-2\times2\end{pmatrix}$, so equation of line is $\mathbf{r}=\begin{pmatrix}-7\\2\\-7\end{pmatrix}+s\begin{pmatrix}-1-(-7)\\2-2\\-1-(-7)\end{pmatrix}=\ldots$	**M1**	3.1a
	$\mathbf{r}=\begin{pmatrix}-7\\2\\-7\end{pmatrix}+s\begin{pmatrix}6\\0\\6\end{pmatrix}$ oe e.g. $\mathbf{r}=\begin{pmatrix}-1\\2\\-1\end{pmatrix}+s\begin{pmatrix}1\\0\\1\end{pmatrix}$	**A1**	2.5
		(2)	
	Alternative 1 to (c) (Not on spec)		
	Normal to Π_2: $\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&-3&5\\10&6&2\end{vmatrix}=\begin{pmatrix}-36\\48\\36\end{pmatrix}$ Direction of l_2: $\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\-3&4&3\\2&3&-2\end{vmatrix}=\begin{pmatrix}-17\\0\\-17\end{pmatrix}$ equation of line is $\mathbf{r}=\begin{pmatrix}-7\\2\\-7\end{pmatrix}+s\begin{pmatrix}-17\\0\\-17\end{pmatrix}$ oe	**M1** **A1**	3.1a 2.5
		(2)	
	Alternative 2 to (c) (Not on spec)		
	Normal to Π_2: $\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&-3&5\\10&6&2\end{vmatrix}=\begin{pmatrix}-36\\48\\36\end{pmatrix}$ $(-3\mathbf{i}+4\mathbf{j}+3\mathbf{k})\cdot(-7\mathbf{i}+2\mathbf{j}-7\mathbf{k})=8$ Π_2 is $3x-4y-3z=-8$ then e.g. solves simultaneously with Π_1 and $x=\lambda$ to give $y=2$, $z=\lambda$ So equation of line is $\mathbf{r}=\begin{pmatrix}0\\2\\0\end{pmatrix}+s\begin{pmatrix}1\\0\\1\end{pmatrix}$ oe	**M1** **A1**	3.1a 2.5
		(2)	
	Alternative 3 to (c) (Not on spec)		
	As alternative 2 to find the equation of plane 2: $3x-4y-3z=-8$ Then solves simultaneously with plane 1 to give e.g. $y=2$, $x=z$ Hence $\mathbf{r}=\begin{pmatrix}s\\2\\s\end{pmatrix}$ oe	**M1** **A1**	3.1a 2.5
		(2)	

Notes:

(c)

M1: A complete method to find the required equation. E.g. finds a second point on the common line and uses this and the point from (a) to find the direction and then forms the vector equation of the required line. May use earlier work to find the second point or start again.

A1: Correct equation formed, accept with any parameter which is not *x*, *y* or *z*.
Must include "**r** =" unless this was already penalised in part (b).

Alternative 1:

M1: Attempts the normal to Π_2 by attempting the vector product of the direction of l_1 and the direction of l_2 and then attempting the vector product of this with the normal to Π_1 to find the direction of the common line and forms the vector equation with a point on the line.
The normal vectors may also be found using scalar products e.g.

$$\mathbf{n} = x\mathbf{i} + y\mathbf{j} + \mathbf{k} \Rightarrow (x\mathbf{i} + y\mathbf{j} + \mathbf{k}) \cdot (\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}) = 0, (x\mathbf{i} + y\mathbf{j} + \mathbf{k}) \cdot (10\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}) = 0 \Rightarrow x = ..., y = ...$$

A1: Correct equation formed, accept with any parameter which is not *x*, *y* or *z*.
Must include "**r** =" unless already penalised in (b).

Alternative 2:

M1: Attempts the normal to Π_2 by attempting the vector product of the direction of l_1 and the direction of l_2 or using scalar products as above and finds the equation of Π_2 by using a point on the line and then solves this with Π_1 by eliminating one of the variables to form the vector equation of the line of intersection.

A1: Correct equation formed, accept with any parameter which is not *x*, *y* or *z*.
Must include "**r** =" unless already penalised in (b).

Alternative 3:

M1: Attempts the normal to Π_2 by attempting the vector product of the direction of l_1 and the direction of l_2 or using scalar products as above and finds the equation of Π_2 by using a point on the line and then solves this simultaneously with Π_1 to form the vector equation of the line of intersection.

A1: Correct equation formed, accept with any parameter which is not *x*, *y* or *z*.
Must include "**r** =" unless already penalised in (b).

There will be other methods in part (c)
Generally the method mark requires a <u>complete</u> attempt
to find the equation of the line of intersection.

(d)	Line from (c) must lie in plane, so $\begin{pmatrix}1\\0\\1\end{pmatrix}$ is perpendicular to $\begin{pmatrix}1\\1\\a\end{pmatrix}$ $\Rightarrow \begin{pmatrix}1\\0\\1\end{pmatrix}.\begin{pmatrix}1\\1\\a\end{pmatrix}=0 \Rightarrow 1\times1+0\times1+1\times a=0 \Rightarrow a=...$	**M1**	3.1a
	$a=-1$	**A1**	1.1b
	$b=\begin{pmatrix}-7\\2\\-7\end{pmatrix}.\begin{pmatrix}1\\1\\-1\end{pmatrix}=2$	**A1**	2.2a
		(3)	
	Alternative 1 to (d):		
	$\begin{pmatrix}-7\\2\\-7\end{pmatrix}.\begin{pmatrix}1\\1\\a\end{pmatrix}=b \Rightarrow -7+2-7a=b$ $\Rightarrow a=...$ or $b=...$ $\begin{pmatrix}-1\\2\\-1\end{pmatrix}.\begin{pmatrix}1\\1\\a\end{pmatrix}=b \Rightarrow -1+2-a=b$	**M1**	3.1a
	$a=-1$ or $b=2$	**A1**	1.1b
	$a=-1$ and $b=2$	**A1**	2.2a
		(2)	
	Alternative 2 to (d) (Not on spec):		
	Normal to Π_2: $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\1 & -3 & 5\\10 & 6 & 2\end{vmatrix}=\begin{pmatrix}-36\\48\\36\end{pmatrix}$ $\begin{vmatrix}3 & -4 & -3\\2 & 3 & -2\\1 & 1 & a\end{vmatrix}=0 \Rightarrow 3(3a+2)+4(2a+2)-3(-1)=0 \Rightarrow a=...$	**M1**	3.1a
	$a=-1$	**A1**	1.1b
	$a=-1$ and $b=2$	**A1**	2.2a
		(2)	
	Alternative 3 to (d):		
	$\mathbf{r}=\begin{pmatrix}0\\2\\0\end{pmatrix}+s\begin{pmatrix}1\\0\\1\end{pmatrix}$ lies in $\Pi_3 \Rightarrow s+2+as=b$ $(a+1)s+2=b \Rightarrow a=...$	**M1**	3.1a
	$a=-1$	**A1**	1.1b
	$a=-1$ and $b=2$	**A1**	2.2a
		(2)	

	Alternative 4 to (d):		
	Normal to Π_2: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & 5 \\ 10 & 6 & 2 \end{vmatrix} = \begin{pmatrix} -36 \\ 48 \\ 36 \end{pmatrix}$ $(-3\mathbf{i}+4\mathbf{j}+3\mathbf{k})\cdot(-7\mathbf{i}+2\mathbf{j}-7\mathbf{k})=8$ Π_2 is $3x-4y-3z=-8$ then solves simultaneously with Π_1 and Π_3 and uses consistency e.g. $3x-4y-3z=-8$ $2x+3y-2z=6$ $x+y+az=b$ $x+y+az=b \Rightarrow x=b-y-az$ $3b-3y-3az-4y-3z=-8 \Rightarrow 7y+(3a+3)z=3b+8$ $2b-2y-2az+3y-2z=6 \Rightarrow y-(2a+2)z=6-2b$ Consistency $\Rightarrow -14a-14=3+3a \Rightarrow a=\ldots$ or $3b+8=42-14b \Rightarrow b=\ldots$	**M1**	3.1a
	$a=-1$ or $b=2$	**A1**	1.1b
	$a=-1$ and $b=2$	**A1**	2.2a

(12 marks)

Notes:

(d)

M1: Realises the direction of (c) is perpendicular to the normal to Π_3 and applies the dot product = 0 to find *a*.

A1: Correct value for *a*

A1: Deduces the value of *b* using their *a* and one of their points on the line.

Alternative 1:

M1: Uses their point from (a) and another point on the line of intersection and substitutes both points into the given equation and solves the resulting equations for *a* or *b*.

A1: Correct value for *a* or *b*.

A1: Correct value for *a* and *b*.

Alternative 2:

M1: Attempts the normal to Π_2 by attempting the vector product of the direction of l_1 and the direction of l_2 or using scalar products as above (may have been found earlier) and then attempts the determinant of the matrix of normal vectors = 0 and solves for *a*

A1: Correct value for *a*.

A1: Deduces the correct value of *b*. E.g. by using their *a* with a point on the line.

Alternative 3:

M1: Substitutes their line from part (c) into the equation for Π_3 and compares coefficients to establish a value for *a*.

A1: Correct value for *a*.

A1: Deduces the correct value of *b*.

Alternative 4:

M1: Finds the equation for Π_2 and then solves all 3 equations using the fact they are consistent leading to a value for *a* or a value for *b*.

A1: Correct value for *a* or *b*.

A1: Correct value for *a* and *b*.

There will be other methods in part (d)
Generally the method mark requires a <u>complete</u> attempt
to find the value of *a* or *b*.