Question	Scheme	Marks	AOs
6(a)	$\frac{dV}{dt}=3-\frac{4}{1+e^{0.8t}}\pm kV$ (where k is constant)	**M1**	3.3
	$t=0, V=10, \frac{dV}{dt}=-3 \Rightarrow -3=3-\frac{4}{1+1}-10k \Rightarrow k=...$	**dM1**	3.4
	$\Rightarrow 10k=4 \Rightarrow k=\frac{2}{5} \Rightarrow \frac{dV}{dt}=3-\frac{4}{1+e^{0.8t}}-0.4V$ *	**A1***	2.1
		(3)	
(b)	$\frac{d}{dt}\left(\arctan e^{0.4t}\right)=\frac{1}{1+\left(e^{0.4t}\right)^2}\times ke^{0.4t}$	**M1**	1.1b
	$\frac{d}{dt}\left(\arctan e^{0.4t}\right)=\frac{2e^{0.4t}}{5\left(1+e^{0.8t}\right)}$ oe	**A1**	1.1b
		(2)	
	Alternative to part (b):		
	$y=\arctan e^{0.4t} \Rightarrow \tan y=e^{0.4t} \Rightarrow \sec^2 y\frac{dy}{dx}=0.4e^{0.4t}$	**M1**	1.1b
	$\frac{dy}{dx}=\frac{0.4e^{0.4t}}{\sec^2 y}=\frac{0.4e^{0.4t}}{1+\tan^2 y}=\frac{0.4e^{0.4t}}{1+\left(e^{0.4t}\right)^2}$	**A1**	1.1b
		(2)	
(c)	$\frac{dV}{dt}+0.4V=3-\frac{4}{1+e^{0.8t}} \Rightarrow I.F.\left(=e^{\int 0.4\,dt}\right)=e^{0.4t}$	**B1**	2.2a
	$e^{0.4t}V=\int 3e^{0.4t}-\frac{4e^{0.4t}}{1+e^{0.8t}}\,dt$	**M1**	1.1b
	$=Ae^{0.4t}-B\arctan\left(e^{0.4t}\right)(+c)$	**M1**	1.1b
	$e^{0.4t}V=7.5e^{0.4t}-10\arctan\left(e^{0.4t}\right)\ (+c)$	**A1**	1.1b
	$V=10, t=0 \Rightarrow 10=7.5-10\arctan 1+c \Rightarrow c=...$	**M1**	3.4
	$V=7.5-10e^{-0.4t}\arctan\left(e^{0.4t}\right)+2.5(\pi+1)e^{-0.4t}$	**A1**	2.1
		(6)	
(d)	E.g. $V(10)\approx 7.4$ litres so the model is not very accurate as it predicts approximately 7.5% below the actual level.	**B1ft**	3.5a
		(1)	
		(12 marks)	

Notes

(a)

M1: Sets up the correct equation for the model using the information in the question.

dM1: Uses the initial conditions to find the constant of proportionality for flow out.

Condone use of $\frac{dV}{dt} = +3$. **Depends on the first mark.**

A1*: Correct equation shown from correct work proceeding via $10k = 4$ to find k.

Attempts in (a) using verification score no marks:

E.g. $\frac{dV}{dt} = 3 - \frac{4}{1+e^{0.8t}} - \frac{2}{5}V \Rightarrow -3 = 3 - \frac{4}{2} - 0.4V \Rightarrow V = \frac{4}{0.4} = 10$

(b)

M1: Differentiates to achieve the form shown. Allow $k = 1$

A1: Correct derivative in any form. Need not be simplified.

Alternative:

M1: Takes tan of both sides and differentiates implicitly and reaches $\frac{1}{1+\left(e^{0.4t}\right)^2} \times ke^{0.4t}$. Allow $k = 1$.

A1: Correct derivative in any form. Need not be simplified.

(c)

B1: Deduces the correct integrating factor for the equation. May be implied by sight of $\frac{d}{dt}\left(e^{0.4t}V\right) = ...$ or equivalent work.

M1: Fully multiplies through by their integrating factor and integrates the LHS (look for $I.F. \times V = \int I.F. \times \left(3 - \frac{4}{1+e^{0.8t}}\right) dt$ though condone missing dt.

M1: Attempts the integral of the RHS.

Award for $\int \alpha e^{0.4t}\, dt = \beta e^{0.4t} \quad \alpha \neq \beta$ or $\int \frac{\alpha e^{0.4t}}{1+e^{0.8t}} dt = \beta \arctan e^{0.4t}, \quad \beta \neq 0$

A1: Correct integration, need not be simplified. Allow if the + c is missing for this mark.

M1: Attempts to find their constant – which must have been treated correctly from point of integration. Note that this is not formally dependent but there must have been an attempt to integrate.

A1: Correct answer. The question says "simplest form" but allow equivalent expressions e.g.

$V = 7.5 - \frac{10\arctan\left(e^{0.4t}\right)}{e^{0.4t}} + \frac{5\pi}{2e^{0.4t}} + \frac{5}{2e^{0.4t}}$ but do not allow inexact values for the constants.

(d)

B1ft: Evaluates V where $V > 0$ at $t = 10$ and makes an appropriate comment.

For the evaluation, allow if a value of V is obtained even if there is no evidence of substitution provided that it is clear that $t = 10$ has not been substituted into something that is not V. So you do not need to check their value.

For the tolerance you may need to use your own judgement but a general guide is:

$0 < V < 7$	Not a good model
$7 \leqslant V < 7.7$ or $8.3 \leqslant V < 9$	Allow good **or** poor model
$7.7 \leqslant V < 8.3$	Good model
$V \geqslant 9$	Not a good model