Question	Scheme	Marks	AOs
8(a)	Accept E.g. • 1 month is too old for “newborn” • The mammals might not start breeding at exactly 3 months old • The mammals will stop breeding beyond a certain age • Being over 3 months old doesn’t necessarily mean the mammal can breed • Some mammals over 3 months may be infertile so will not be breeders • Some juveniles might be breeders But not • The size of the categories is different • There might be overlap • The exact age of mammals might not be known • The numbers in each category will be different • Breeding age is different for different species	**B1**	3.5b
		(1)	

(a)

B1: Any valid limitation – see scheme for some examples. Must refer to a feature of the categories given.

(b)(i)	$\begin{pmatrix}0&0&2\\a&b&0\\0&0.48&0.96\end{pmatrix}^2\begin{pmatrix}0\\0\\k\end{pmatrix}=\begin{pmatrix}0&0&2\\a&b&0\\0&0.48&0.96\end{pmatrix}\begin{pmatrix}2k\\0\\0.96k\end{pmatrix}$ or $\begin{pmatrix}0&0.96&1.92\\ab&b^2&2a\\0.48a&0.48b+0.96\times0.48&0.96^2\end{pmatrix}\begin{pmatrix}0\\0\\k\end{pmatrix}=\begin{pmatrix}1.92k\\2ak\\0.9216k\end{pmatrix}$	**M1**	3.4
	$48=2\times0.96k\Rightarrow k=...$	**dM1**	1.1b
	$k=25$ so 25 mammals at the start of the study	**A1**	3.2a
(ii)	$40=2ka\Rightarrow a=0.8*$	**A1***	1.1b
		(4)	

(b)(i)

M1: Attempts to use the given information to set up a matrix equation and find the numbers of mammals after one month e.g.

$$\begin{pmatrix}0&0&2\\a&b&0\\0&0.48&0.96\end{pmatrix}\begin{pmatrix}N_0\\J_0\\B_0\end{pmatrix}=\begin{pmatrix}2B_0\\aN_0+bJ_0\\0.48J_0+0.96B_0\end{pmatrix}$$

or attempts to square the matrix to find the number of mammals after two months e.g.

$$\begin{pmatrix}0&0.96&1.92\\ab&b^2&2a\\0.48a&0.48b+0.96\times0.48&0.96^2\end{pmatrix}\begin{pmatrix}N_0\\J_0\\B_0\end{pmatrix}=...$$

dM1: Forms an equation, in their variable for number of breeders at the start, setting their number of newborns after 2 months equal to 48 and solves for their variable to find the initial number of breeders.

A1: For identifying 25 mammals at the start of the study. Allow 25 mammals or just 25 or e.g. $B_0=25$ so ignore how they label it just look for 25

Note that in some cases work may be minimal e.g.

$$\begin{pmatrix}0&0&2\\a&b&0\\0&0.48&0.96\end{pmatrix}\begin{pmatrix}0\\0\\B_0\end{pmatrix}=\begin{pmatrix}N_1\\J_1\\B_1\end{pmatrix}\Rightarrow0.96B_0=B_1,\ \begin{pmatrix}0&0&2\\a&b&0\\0&0.48&0.96\end{pmatrix}\begin{pmatrix}N_1\\J_1\\B_1\end{pmatrix}=\begin{pmatrix}48\\40\\B_2\end{pmatrix}\Rightarrow2B_1=48$$

$$B_1=24=0.96B_0\Rightarrow B_0=25$$

(ii)

A1*: For correctly showing *a* = 0.8. Must see the correct work to establish the correct value or equivalent by verification with a minimal conclusion e.g.

$$\begin{pmatrix}0&0&2\\0.8&b&0\\0&0.48&0.96\end{pmatrix}\begin{pmatrix}0\\0\\25\end{pmatrix}=\begin{pmatrix}50\\0\\24\end{pmatrix}=\begin{pmatrix}48\\40\\B_2\end{pmatrix}\Rightarrow0.8\times50=40\ \text{Hence true *}$$

(c)	$\det\begin{pmatrix}0 & 0 & 2\\ 0.8 & b & 0\\ 0 & 0.48 & 0.96\end{pmatrix} = 0-0+2(0.48\times 0.8-0) = 0.768$	**B1**	2.2a
	$\text{adj}\begin{pmatrix}0 & 0 & 2\\ 0.8 & b & 0\\ 0 & 0.48 & 0.96\end{pmatrix} = \begin{pmatrix}0.96b & 0.96 & -2b\\ -0.768 & 0 & 1.6\\ 0.384 & 0 & 0\end{pmatrix}$	**M1**	1.1b
	$= \begin{pmatrix}1.25b & 1.25 & -\frac{125}{48}b\\ -1 & 0 & \frac{25}{12}\\ 0.5 & 0 & 0\end{pmatrix}$ oe e.g. $\frac{1}{0.768}\begin{pmatrix}0.96b & 0.96 & -2b\\ -0.768 & 0 & 1.6\\ 0.384 & 0 & 0\end{pmatrix}$ or e.g. $\frac{125}{96}\begin{pmatrix}0.96b & 0.96 & -2b\\ -0.768 & 0 & 1.6\\ 0.384 & 0 & 0\end{pmatrix}$	**A1**	1.1b
		(3)	

(c)

B1: Deduces correct determinant for the matrix. Allow equivalents e.g. $\frac{96}{125}$ May be implied.

M1: Recognisable attempt at the adjoint matrix. Look for at least 3 non-zero entries correct.

A1: Correct inverse. Accept awrt $-2.6b$ for the upper right entry and awrt 2.08 for middle right entry, or accept with determinant still outside. Apply isw once a correct answer is seen.

(d)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1.25b & 1.25 & -\frac{125}{48}b \\ -1 & 0 & \frac{25}{12} \\ 0.5 & 0 & 0 \end{pmatrix}\begin{pmatrix} 596 \\ 464 \\ 437 \end{pmatrix}$ $\text{Total} = 1.25b \times 596 + 1.25 \times 464 - \frac{125}{48}b \times 437 - 596 + \frac{25}{12} \times 437 + 0.5 \times 596$	**M1**	3.1b
	$\Rightarrow 1015 = x + y + z = 745b + 580 - 1138b - 596 + 910.4 + 298 \Rightarrow b = ...$	**dM1**	3.4
	$b = \text{awrt } 0.45$	**A1**	1.1b
		(3)	
	(d) Alternative:		
	$\begin{pmatrix} 0 & 0 & 2 \\ 0.8 & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 596 \\ 464 \\ 437 \end{pmatrix} \Rightarrow \begin{matrix} 2z = 596 \\ 0.8x + by = 464 \\ 0.48y + 0.96z = 437 \end{matrix}$ $\Rightarrow z = 298, y = \frac{3773}{12}(314.4...)$ $x + y + z = 1015 \Rightarrow x = ...\frac{4831}{12}(402.5...)$	**M1**	3.1b
	$0.8x + by = 464 \Rightarrow 0.8 \times \frac{4831}{12} + b \times \frac{3773}{12} = 464 \Rightarrow b = ...$	**dM1**	3.4
	$b = \text{awrt } 0.45$	**A1**	1.1b
		(3)	

(d)

M1: Attempts (their inverse matrix) $\times \begin{pmatrix} 596 \\ 464 \\ 437 \end{pmatrix}$ correctly and adds the 3 expressions together to find the total in terms of *b*.

M1: Sets their total = 1015 and solves for *b*.

A1: awrt 0.45

Alternative:

M1: Uses the original matrix with $a = 0.8$ and $\begin{pmatrix} 596 \\ 464 \\ 437 \end{pmatrix}$ to form 3 equations in their variables and *b* and uses these and the 1015 to find the number of Newborns.

M1: Uses their values in the *y* component and solves for *b*.

A1: awrt 0.45

(e)	Let NM_n be newborn males and NF_n be newborn females in month n $$\begin{pmatrix} NM_{n+1} \\ NF_{n+1} \\ J_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0.84 \\ 0 & 0 & 0 & 1.16 \\ ? & ? & 0.45 & 0 \\ 0 & 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} NM_n \\ NF_n \\ J_n \\ B_n \end{pmatrix}$$ or e.g. $$\begin{pmatrix} NF_{n+1} \\ NM_{n+1} \\ J_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1.16 \\ 0 & 0 & 0 & 0.84 \\ ? & ? & 0.45 & 0 \\ 0 & 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} NF_n \\ NM_n \\ J_n \\ B_n \end{pmatrix}$$	**M1** **A1ft**	3.5c 3.3
		(2)	
			(13 marks)

Notes:

(e)

M1: Defines new variables for male and female newborns (accept if a clear notation is used if not defined) and sets up a 4×4 matrix with structure shown, or male and female rows swapped, with the correct 0 entries in at least 4 places.

A1ft: Fully correct matrix system shown, accepting anything (including 0) for the unknown spaces shown – but must have all the 0's **and** upper right entries correct. Accept *b* or their value of *b* in place of 0.45