Question	Scheme	Marks	AOs
3(a)	Max $r=4+a=5.5\Rightarrow a=...$	M1	3.4
	$a=1.5$	A1	2.2a
		(2)	
(b)	$\text{Pool area}=\frac{1}{2}\int_0^{2\pi}(4-\text{"1.5"}\sin 3\theta)^2\,d\theta$	M1	3.1a
	$(4-\text{"1.5"}\sin 3\theta)^2=16-\text{"12"}\sin 3\theta+\text{"2.25"}\sin^2 3\theta$ $=16-\text{"12"}\sin 3\theta+\text{"2.25"}\left(\frac{1-\cos 6\theta}{2}\right)\left[=16-8a\sin 3\theta+a^2\left(\frac{1-\cos 6\theta}{2}\right)\right]$	M1	2.1
	$\int(4-\text{"1.5"}\sin 3\theta)^2\,d\theta=16\theta+\text{"4"}\cos 3\theta+\text{"}\frac{9}{8}\text{"}\left(\theta-\frac{\sin 6\theta}{6}\right)$ $\left[=16\theta+\frac{8a}{3}\cos 3\theta+\frac{a^2}{2}\left(\theta-\frac{\sin 6\theta}{6}\right)\right]$	A1ft	1.1b
	$\frac{1}{2}\left[\text{"}\frac{137}{8}\text{"}\theta+\text{"4"}\cos 3\theta-\text{"}\frac{3}{16}\text{"}\sin 6\theta\right]_0^{2\pi}=...\left(\frac{137}{8}\pi\right)$	dM1	3.1a
	Area of $T=\pi\times 36-\frac{137}{8}\pi$	DM1	1.1b
	$=\frac{151}{8}\pi\ (\text{m}^2)$ oe	A1	1.1b
		(6)	
		(8 marks)	

Notes

(a)

M1: Uses all the information given for the model and realises the maximum value of r is $(4 + a)$ and uses the radius of the circle to find a value for a.

A1: Deduces the correct value of a. Note $a=-1.5$ can potentially gain M1A0.

(b)

Note accept with their a, a made up a or even a itself for the first 5 marks. Note use of $a=-1.5$ can score full marks.

M1: Adopts a correct strategy for the area of the pool. This requires the correct use of the polar area formula including the ½ .

Note the ½ may be implied by choice of limits (e.g. any span of π radians without the half implies an attempt at doubling so the ½ may not appear (even if the symmetry is incorrect).)

M1: Squares the bracket, achieving three terms, and applies $\sin^2 3\theta=\frac{\pm 1\pm\cos 6\theta}{2}$ in order to reach an integrable form. Condone numerical slips when expanding.

A1ft: Correct integration in any form (½ not needed here) (follow through their a). ie as shown in scheme or if gathered it is $(32+a^2)\frac{\theta}{2}+\frac{8a}{3}\cos 3\theta-a^2\frac{\sin 6\theta}{12}$

dM1: Depends on previous M. Uses appropriate limits for their integrated function. Allow limits other than 0 and 2π by using symmetry provided the correct multiple is used, e.g. $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ followed by doubling (which may be cancelled with the $\frac{1}{2}$) or $-\frac{\pi}{6}$ and $\frac{\pi}{2}$ and multiplying by 3

DM1: Depends on first M. Fully correct strategy for obtaining the area of T. Must have a correct attempt at the circle area (maybe be via integration) and area inside the curve. Symmetry may have been used.

A1: Correct area from fully correct work (all previous marks scored). Units not required. The decimal answer 18.875π is acceptable, but answer must be exact, not a rounded decimal.

Note: $\sin^2 3\theta = \frac{\pm 1 \pm \cos 2\theta}{2}$ use will score a maximum of M1M0A0dM0DM1A0

Note: if $\frac{1}{2}\int_0^{2\pi} 6^2 - (4 - a\sin 3\theta)^2 \, \mathrm{d}\theta$ is used the scheme will follow as above with A1ft for $10\theta - \frac{8a}{6}\cos 3\theta - \frac{a^2}{4}\left(\theta - \frac{\sin 6\theta}{6}\right)$ and dM1DM1 gained together.

Note: Integrating over 0 to 2π means the trig terms will disappear so watch out for incorrect trig terms in the integration, which will lead to the correct answer but will lose both A marks.