Question	Scheme	Marks	AOs
4(a)	$z^n + \frac{1}{z^n} \equiv e^{in\theta} + \frac{1}{e^{in\theta}} \equiv e^{in\theta} + e^{-in\theta}$	M1	1.1b
	$\equiv \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta \equiv 2\cos n\theta$ *	A1*	2.1
		(2)	
(b)	$\left(z+z^{-1}\right)^5 = 32\cos^5\theta$	B1	2.2a
	$\left(z+z^{-1}\right)^5 = z^5 + 5z^3 + 10z + 10z^{-1} + 5z^{-3} + z^{-5}$	M1 A1	1.1b 1.1b
	$32\cos^5\theta = \left(z^5+z^{-5}\right) + 5\left(z^3+z^{-3}\right) + 10\left(z+z^{-1}\right)$ $= 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$	M1	2.1
	$\cos^5\theta = \frac{1}{16}\left(\cos 5\theta + 5\cos 3\theta + 10\cos\theta\right)$ *	A1*	1.1b
		(5)	
(c)	$\cos 5\theta + 5\cos 3\theta + 10\cos\theta = -2\cos\theta \Rightarrow 16\cos^5\theta = -2\cos\theta$	B1	3.1a
	$2\cos\theta\left(8\cos^4\theta + 1\right) = 0 \Rightarrow \theta = \ldots$	M1	1.1b
	$8\cos^4\theta + 1 = 0$ has no solution so $\cos\theta = 0$ $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$	A1	2.2a
		(3)	
		(10 marks)	

Notes

(a)

M1: Substitutes *z* into the LHS and simplifies the powers as shown. Allow if they go direct to trigonometric expressions without exponentials. The mark is for sorting out the negative index.

A1*: Converts the exponential form to trigonometric form correctly and correctly completes the proof with no errors seen. The trigonometric expansion must be clearly seen. Condone missing brackets in e.g $\cos -n\theta$ terms if intent is clear. Note the LHS of the identity may be implied.

(b)

B1: Deduces that $\left(z+z^{-1}\right)^5 = 32\cos^5\theta$ Do not accept 2^5 for 32. May be implied.

M1: Attempts to expand $\left(z+z^{-1}\right)^5$. Correct binomial coefficients must be used, terms need not be simplified. Condone at most one slip in powers.

A1: Correct expansion, terms need not be gathered but powers must have been simplified.

M1: Sets their expressions equal and applies the result from (a) – grouping must be shown.

A1*: Reaches the printed answer with no errors and relevant steps all shown.

(c)

B1: Uses the result from (b) to deduce the correct equation.

M1: Must have attempted to use part (b) to obtain $\alpha\cos^5\theta = \beta\cos\theta$ or equivalent. Collects to one side and attempts to factorise and solve. Note dividing through by cos θ is M0.

A1: Rejects the inappropriate solution and selects $\cos\theta = 0$ and obtains the correct values only. The equation must have been correct. There must have been some consideration of the $8\cos^4\theta + 1$ e.g. stating $8\cos^4\theta > 0$ so no solutions, or attempting to find complex roots and deducing no answers. May be minimal, but some consideration that no roots arise from this part must have been given.

Note: The correct answer will appear from incorrect attempts – the M must be gained in order to award the A. E.g. assuming the equation reduces to $\cos^5\theta = 0$ will score B0M0A0. Likewise, answers only scores B0M0A0 (questions says hence so use of (b) must be seen).

Alt (a)	$z^n+\frac{1}{z^n}=\cos n\theta+\mathrm{i}\sin n\theta+\frac{1}{\cos n\theta+\mathrm{i}\sin n\theta}=\frac{\cos^2 n\theta+2\mathrm{i}\cos n\theta\sin n\theta-\sin^2 n\theta+1}{\cos n\theta+\mathrm{i}\sin\theta}$	M1	1.1b
	$=\frac{2\cos^2 n\theta+2\mathrm{i}\cos n\theta\sin n\theta}{\cos n\theta+\mathrm{i}\sin n\theta}=\frac{2\cos n\theta\left(\cos n\theta+\mathrm{i}\sin n\theta\right)}{\cos n\theta+\mathrm{i}\sin n\theta}=2\cos n\theta$ *	A1*	2.1
		(2)	

Notes

Alt: by De Moivre
M1: Applies De Moivre on both terms and puts over a common denominator.
A1*: Complete correctly, using $1-\sin^2 n\theta=\cos^2 n\theta$ and cancelling $\cos n\theta+\mathrm{i}\sin n\theta$. No errors seen.

Alt (b)	$\cos 5\theta=\mathrm{Re}\left(\cos 5\theta+\mathrm{i}\sin 5\theta\right)=\mathrm{Re}\left(\cos\theta+\mathrm{i}\sin\theta\right)^5$	B1	2.2a
	$\left(\cos\theta+\mathrm{i}\sin\theta\right)^5=c^5+5\mathrm{i}c^4s+10\mathrm{i}^2c^3s^2+10\mathrm{i}^3c^2s^3+5\mathrm{i}^4cs^4+\mathrm{i}^5s^5$	M1	1.1b
	$\mathrm{Re}\left(\cos\theta+\mathrm{i}\sin\theta\right)^5=\cos^5\theta-10\cos^3\theta\sin^2\theta+5\cos\theta\sin^4\theta$	A1	1.1b
	$\cos 5\theta=\cos^5\theta-10\cos^3\theta\left(1-\cos^2\theta\right)+5\cos\theta\left(1-\cos^2\theta\right)^2$ $=16\cos^5\theta-20\cos^3\theta+5\cos\theta$ $=16\cos^5\theta-\frac{20}{4}\left(\cos 3\theta+3\cos\theta\right)+5\cos\theta$	M1	2.1
	$\Rightarrow\cos^5\theta=\frac{1}{16}\left(\cos 5\theta+5\cos 3\theta+10\cos\theta\right)$*	A1*	1.1b
		(5)	

Notes

Alt: by De Moivre
B1: Correctly stated or clearly implied De Moivre statement for $\cos 5\theta$
M1: Attempts to expand $\left(\cos\theta+\mathrm{i}\sin\theta\right)^5$ Correct coefficients but allow one slip per main scheme.
The powers of i need not be simplified for the attempt at expansion, accept if only the real terms are shown. Allow c and s notation.
A1: Correct real terms extracted with the i's removed.
M1: Applies $\sin^2\theta=1-\cos^2\theta$ to reduce to an equation in $\cos\theta$ **and** applies $\cos^3\theta=\frac{1}{4}\left(\cos 3\theta+3\cos\theta\right)$ (quoted or derived – allow a slip if derived) to get to an equation without powers of cos terms.
A1*: Reaches the printed answer with no errors and relevant steps all shown.