Question	Scheme	Marks	AOs
5(a)	$(t+2)\dfrac{\mathrm{d}v}{\mathrm{d}t}+3v=k(t+2)-3 \Rightarrow \dfrac{\mathrm{d}v}{\mathrm{d}t}+\dfrac{3v}{t+2}=k-\dfrac{3}{t+2}$ $I=\mathrm{e}^{\int\frac{3}{t+2}\mathrm{d}t}=(t+2)^3$	M1	3.1b
	$\Rightarrow v(t+2)^3=\int\left(k(t+2)^3-3(t+2)^2\right)\mathrm{d}t=...$	M1	1.1b
	$v(t+2)^3=\dfrac{k}{4}(t+2)^4-(t+2)^3(+c)$ $\left[\text{Alt } (t+2)^3v=\dfrac{kt^4}{4}+2kt^3-t^3+6kt^2-6t^2+8kt-12t(+c)\right]$	A1	1.1b
	$t=0,\ v=0 \Rightarrow c=8-4k$	M1	3.4
	$v(t+2)^3=\dfrac{k}{4}(t+2)^4-(t+2)^3+8-4k$ $\Rightarrow v=\dfrac{k}{4}(t+2)-1+\dfrac{4(2-k)}{(t+2)^3}*$	A1*	2.1
		(5)	
(b)	$v=4,\ t=2 \Rightarrow 4=k-1+\dfrac{4(2-k)}{64} \Rightarrow k=...(5.2)$	M1	3.1b
	$\Rightarrow v=\dfrac{"5.2"}{4}(5+2)-1+\dfrac{4(2-"5.2")}{(5+2)^3}=...$	dM1	3.4
	Velocity is $8.06\,\mathrm{ms}^{-1}$ (awrt)	A1	3.2a
		(3)	
(c)	E.g. • The model suggests the velocity increases indefinitely which is unlikely • The raindrop will reach the ground/doesn't fall forever so not valid • The raindrop will reach a terminal velocity after a finite time so not good model	B1	3.5a
		(1)	
			(9 marks)

Notes

(a)

M1: Begins to solve the problem with a correct process to find the integrating factor for the model. Look for an attempt at $e^{\int \frac{A}{t+2}\,dt} = ...$ where A is a constant (positive or negative). For attempts at IF by recognition they must achieve form $A(t+2)^3$

M1: Attempts the solution of the differential equation using a correct method with their integrating factor. The dt may be implied. They must attempt the integration as part of the method, though it need not be correct – any changed function will do. So score for $v \times \text{their IF} = \text{their attempt at integrating } \text{their IF} \times \left(k - \frac{3}{t+2}\right)$ allowing for errors expanding this bracket.

A1: Correct solution. The + c may be missing for this mark.

M1: Interprets the initial conditions to find the constant of integration – must have a constant of integration for this mark and the correct initial conditions must be clear.

A1*: Obtains the printed answer with no errors and at least one intermediate line between $v(t+2)^3 = \frac{k}{4}(t+2)^4 - (t+2)^3 + c$ and the stated answer. Allow terms in different order, but the $8-4k$ must be factorised.

(b)

M1: Uses the given conditions to establish the value of k in the model

dM1: Uses their value of k and the equation of the model with $t = 5$ to find a value for v. If a value of k has been found accept "$t = 5$ $v =$…" with any value for an attempt.

A1: Correct value **including units** (allow awrt 8.06 ms^{-1}). Accept exact answer 5531/686

(c)

B1: Evaluates the model by making a suitable comment – see scheme for examples. Accept answers relating to the size of velocity increasing beyond what is reasonable or answers that allude to the situation not being able to continue for ever. Accept if symbols are used, e.g. "as $t \to \infty, v \to \infty$ which cannot happen" but must include a comment on validity, not just a statement about what happens (e.g. $t \to \infty, v \to \infty$ with no comment).

Do not accept e.g. "v does not tend to a value" as this is just a statement about what happens. If contrary information is given then award B0.

Some examples of borderline answers seen.

"Not valid as rain is limited to how fast its velocity is therefore can't be big." Scores B1

"Due to gravitational force and not huge height between cloud and ground, the model would be inaccurate for large period of time, because the velocity of raindrops would be a big number, which is unlikely." B1

"Not valid for very large value of t as raindrop will not move that fast." B0

"Raindrop has very little mass so effect due to gravity is lower. Unlikely to have a 1000ms^{-1} raindrop. Not very valid model for large values of t" B1

"The water droplet wouldn't always follow the model in cases where it hits a surface" B0

"No valid as raindrop will hit ground soon. Speed can be infinitely big which is not plausible." B1

"Would not be valid as the raindrop likely has already hit the ground." B1

You may need to use your judgement on whether their comments are acceptable.