Question	Scheme	Marks	AOs
6	If $n=1$ $\sum_{r=1}^{n}(2r-1)^2=(2-1)^2=1$ and $\frac{1}{3}n(4n^2-1)=\frac{1}{3}(1)(4(1)^2-1)=1$ **(LHS=RHS)** so true for $n = 1$	B1	2.4
	(Assume true for $n = k$ so $\sum_{r=1}^{k}(2r-1)^2=\frac{1}{3}k(4k^2-1)$ then) $\sum_{r=1}^{k+1}(2r-1)^2=\frac{1}{3}k(4k^2-1)+(2(k+1)-1)^2$	M1	2.1
	E.g $=\frac{1}{3}(2k+1)(2k^2+5k+3)$ or $=\frac{4}{3}k^3-\frac{1}{3}k+4k^2+4k+1$	dM1	1.1b
	$\frac{1}{3}(k+1)(2k+3)(2k+1)$ or $\frac{1}{3}(k+1)(4k^2+8k+3)$ or $\frac{4k^3}{3}+4k^2+\frac{11k}{3}+1$	A1	1.1b
	$=\frac{1}{3}(k+1)(4(k+1)^2-1)$ Or see notes	A1	2.2a
	If the statement is true for $n = k$ then it has been shown true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all positive integers n.	A1	2.4
		(6)	
			(6 marks)

Notes

B1: Demonstrates the statement is true for $n = 1$. Accept as minimum $(1)^2=1$ and $\frac{1}{3}(3)=1$ for the check (both sides must clearly be evaluated as 1), and for the conclusion "true for $n=1$" or broken "if/when $n=1$ <check> (hence) true/shown/tick" – must be part of the initial check (not just in the conclusion).

M1: Assume the result for $n = k$ and attempts to add the $(k + 1)^{th}$ term to the result for the sum to k terms. An assumption may be clearly made, but accept tacit assumptions. Allow if there are minor slips in the expressions if the intent is clear.

dM1: Makes progress towards proving the inductive step by either factorising the common factor $(2k+1)$ or expanding fully. There may be variations so score for equivalent progress.

A1: Either for a correct factorised form with $(k + 1)$ as a factor or a fully correct expansion with gathered terms. Must have come from correct work from the inductive step, not backwards worked from the $n = k + 1$ expression.

A1: Completes the inductive steps by obtaining the correct expression in terms of $(k + 1)$ with suitable intermediate step e.g. clear factorisation of $(k+1)$ first, OR by expanding the required expression for $n = k + 1$ to achieve equal expressions. Must have been completely correct work, no errors.

A1: Depends on the MMAA marks having been scored with at least an attempt to check one side of the $n = 1$ case having been made. Correct complete conclusion. Must include the notions of "true for $n = 1$", "true for $n = k$ implies true for $n = k+1$" and "hence true for all n" though the exact wording will vary. The conclusion should be given at the end with the exception that the "true for $n = 1$" may be stated with the initial check.

Note: stating "true for $n = k$ and $n = k + 1$" will be A0.

Note: Accept use of n instead of k for all except the final A mark.

Note: Attempts at using summation formulae without induction score no marks.