Question	Scheme	Marks	AOs
7(a)	$(3\mathbf{i}-10\mathbf{j}-\mathbf{k})\cdot(2\mathbf{i}+\mathbf{j}-4\mathbf{k})=6-10+4=0$ $(3\mathbf{i}-10\mathbf{j}-\mathbf{k})\cdot(6\mathbf{i}+\mathbf{j}+8\mathbf{k})=18-10-8=0$	M1	1.1b
	So $3\mathbf{i}-10\mathbf{j}-\mathbf{k}$ is perpendicular to Π	A1	2.1
		(2)	
(b)	$(3\mathbf{i}-10\mathbf{j}-\mathbf{k})\cdot(\mathbf{i}-2\mathbf{j}+3\mathbf{k})=3+20-3=20$	M1	1.1b
	$3x-10y-z=20$	A1	2.5
		(2)	
(c)	$(3\mathbf{i}-10\mathbf{j}-\mathbf{k})\cdot(5\mathbf{i}+p\mathbf{j}-7\mathbf{k})=\text{"}20\text{"}$ $\Rightarrow 15-10p+7=\text{"}20\text{"}\Rightarrow p=...$	M1	3.1a
	$p=0.2$ (oe)	A1	1.1b
		(2)	
(d)	E.g. $1+2\lambda=5+6\mu,\ 3-4\lambda=-7+8\mu\Rightarrow\lambda=...$ or $\mu=...$ $\mu=0.1$ (or $\lambda=2.3$) $\Rightarrow A(5.6,\ 0.3,-6.2)$	M1	1.1b
	$12\mathbf{i}-11\mathbf{j}+6\mathbf{k}-(5.6\mathbf{i}+0.3\mathbf{j}-6.2\mathbf{k})=6.4\mathbf{i}-11.3\mathbf{j}+12.2\mathbf{k}$ $(6.4\mathbf{i}-11.3\mathbf{j}+12.2\mathbf{k})\cdot(3\mathbf{i}-10\mathbf{j}-\mathbf{k})=19.2+113-12.2=120$	M1	3.1a
	$120=\sqrt{6.4^2+11.3^2+12.2^2}\sqrt{3^2+10^2+1^2}\cos\alpha\Rightarrow\alpha=...$ or e.g. $120=\sqrt{6.4^2+11.3^2+12.2^2}\sqrt{3^2+10^2+1^2}\sin\alpha\Rightarrow\alpha=...$	M1	1.1b
	Angle between *AB* and plane $\theta=40°$ (awrt)	A1	1.1b
		(4)	
		(10 marks)	

Notes

(a)
M1: Attempts the scalar product between the given vector and the 2 direction vectors (calculation should be shown for at least one).
A1: Obtains zero for both with sufficient working shown and concludes perpendicular (each one or states both are). Accept normal to as equivalent to perpendicular to.
(b)
M1: Attempts the scalar product between the normal vector and $\mathbf{i}-2\mathbf{j}+3\mathbf{k}$. Allow if the vector product is used again from scratch. Allow equivalent approaches such as using the normal to form the equation $3x-10y-z=d$ and substituting (1, −2, 3) to find d. Maybe be implied by a correct equation if no incorrect working is seen.
A1: Correct equation in Cartesian form (allow equivalent Cartesian equations).
(c)
M1: Attempts the scalar product between the normal vector and $5\mathbf{i}+p\mathbf{j}-7\mathbf{k}$ (or any point on the line), sets = their 20 and solves a linear equation in p. Alternatively substitutes $x=5$, $y=p$ and $z=-7$ into their Cartesian equation and solves for p, or any other complete method to find p.
A1: For $p=0.2$ (oe)
(d)
M1: A complete method to find the coordinates of *A*. Allow if a slip in transcribing a coordinate occurs – it is a mark for the method. If the correct vectors are clearly intended, give credit.
M1: Attempts the vector *AB* and attempts the scalar product with this and the normal vector. Note that if the correct formulae are seen then allow the method for any value appearing afterwards.
M1: Complete method to find the required angle or its complement.
A1: For awrt 40°

Note: Accept alternative vector notations throughout. Where method is not shown for finding coordinates, it may be implied by two correct coordinates.			
Alt (a)	$(2\mathbf{i}+\mathbf{j}-4\mathbf{k})\times(6\mathbf{i}+\mathbf{j}+8\mathbf{k})=(8+4)\mathbf{i}-(16+24)\mathbf{j}+(2-6)\mathbf{k}$	M1	1.1b
	$=12\mathbf{i}-40\mathbf{j}-4\mathbf{k}=4(3\mathbf{i}-10\mathbf{j}-\mathbf{k})$ So $3\mathbf{i}-10\mathbf{j}-\mathbf{k}$ is perpendicular to Π	A1	2.1
		(2)	
Notes			
M1: Attempts the vector product between the direction vectors of the lines, allowing at most one slip in the brackets. A1: Obtains the correct vectors and shows it is a multiple of the required answer and concludes perpendicular.			
Alt II (a)	Normal to plane is $\mathbf{n}=\alpha\mathbf{i}+\beta\mathbf{j}+\gamma\mathbf{k}\Rightarrow(\alpha\mathbf{i}+\beta\mathbf{j}+\gamma\mathbf{k})\bullet(2\mathbf{i}+\mathbf{j}-4\mathbf{k})=0$ and $(\alpha\mathbf{i}+\beta\mathbf{j}+\gamma\mathbf{k})\bullet(6\mathbf{i}+\mathbf{j}+8\mathbf{k})=0$ So $2\alpha+\beta-4\gamma=0,\ 6\alpha+\beta+8\gamma=0$ $\alpha=3\Rightarrow\beta=4\gamma-6,\beta=-8\gamma-18\Rightarrow\beta=..,\gamma=..$	M1	1.1b
	$\gamma=-1,\beta=-10\Rightarrow 3\mathbf{i}-10\mathbf{j}-\mathbf{k}$ perpendicular to Π	A1	2.1
		(2)	
Notes			
M1: Sets up a normal vector to the plane in terms of variables and attempts the vector product between the direction vectors of the lines and the normal to form simultaneous equations and attempts to solve by choosing one of the values and solving for the other two. A1: Correct values, and obtains the correct vector, or a multiple of it and shows it is a multiple of the required answer and concludes perpendicular.			

Alt (d)	E.g. $1+2\lambda=5+6\mu,\ 3-4\lambda=-7+8\mu\Rightarrow\lambda=...$ or $\mu=...$ $\mu=0.1$ (or $\lambda=2.3$) $\Rightarrow A(5.6,\ 0.3,-6.2)$	M1	1.1b
	$\overrightarrow{BX}=m\mathbf{n}\Rightarrow\overrightarrow{OX}=\overrightarrow{OB}+m\mathbf{n}=(12-3m)\mathbf{i}-(11+10m)\mathbf{j}+(6-m)\mathbf{k}$ $\Rightarrow 3(12-3m)-10(-11-10m)-(6-m)=20\Rightarrow m=...\left(-\frac{12}{11}\right)$ $\left[\Rightarrow X=\left(\frac{96}{11},-\frac{1}{11},\frac{78}{11}\right)\right]$	M1	3.1a
	$\sin\alpha=\frac{BX}{AB}=\frac{\frac{12}{11}\times\sqrt{3^2+10^2+1^2}}{\sqrt{6.4^2+11.3^2+12.2^2}}\Rightarrow\alpha=...$	M1	1.1b
	Angle between AB and plane $\theta=40°$ (awrt)	A1	1.1b
		(4)	
Notes			
M1: A complete method to find the coordinates of A. M1: Forms equation for $\overrightarrow{OX}=\overrightarrow{OB}+m\mathbf{n}$ and proceeds to find where normal to plane through B intersects the plane, or the distance BX ($=m\lvert\mathbf{n}\rvert$) M1: Complete method to find the required angle or its complement. May find X first to find BX etc, or may use different sides of the triangle. A1: For awrt 40°			