Question	Scheme	Marks	AOs
8(a)	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=\frac{\mathrm{d}x}{\mathrm{d}t}+\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}x}{\mathrm{d}t}+3y-2x$	M1	1.1b
	$=\frac{\mathrm{d}x}{\mathrm{d}t}+3\left(\frac{\mathrm{d}x}{\mathrm{d}t}-x\right)-2x$	M1	2.1
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}-4\frac{\mathrm{d}x}{\mathrm{d}t}+5x=0$ *	A1*	1.1b
		(3)	
(b)	$m^2-4m+5=0\Rightarrow m=...$	M1	3.4
	$m=2\pm\mathrm{i}$	A1	1.1b
	$x=\mathrm{e}^{\alpha t}\left(A\cos\beta t+B\sin\beta t\right)$	M1	3.4
	$x=\mathrm{e}^{2t}\left(A\cos t+B\sin t\right)$	A1	1.1b
		(4)	
(c)	$y=\frac{\mathrm{d}x}{\mathrm{d}t}-x=\mathrm{e}^{2t}\left(B\cos t-A\sin t+2A\cos t+2B\sin t\right)-\mathrm{e}^{2t}\left(A\cos t+B\sin t\right)$	M1	3.4
	$y=\mathrm{e}^{2t}\left(\left(A+B\right)\cos t+\left(B-A\right)\sin t\right)$	A1	1.1b
		(2)	
(d)	$A=100,\ 275=A+B\Rightarrow B=175$	M1	3.3
	$x=y\Rightarrow 100\cos t+175\sin t=275\cos t+75\sin t\Rightarrow\tan t=...$	dM1	3.1a
	$\tan t=1.75$	A1	1.1b
	$T=24\tan^{-1}\left(1.75\right)=...$	M1	3.2a
	= 25.24	A1	1.1b
		(5)	
(e)	E.g. • Both populations become negative for some times which is not possible	B1	3.5b
		(1)	
			(15 marks)

Notes

Note: Accept the dot notation $\dot{x}$ etc, as equivalents for the derivatives wrt t throughout.

(a)

M1: Differentiates the first equation and substitutes for $\frac{\mathrm{d}y}{\mathrm{d}t}$ from the second equation.

M1: Proceeds to an equation in x and $\frac{\mathrm{d}x}{\mathrm{d}t}$ only by substituting for y.

A1*: Achieves the printed answer with no errors.

(b)

M1: Uses the model to form and solve the Auxiliary Equation.

A1: Correct roots of the AE.

M1: Uses the model to form a Complementary Function appropriate for their roots. Accept complex index form.

A1: Correct General Solution. Accept complex index form. Must include the x

(c)
M1: Uses the model and their answer to part (b) to give y in terms of t. Must involve an attempt at the product rule. Alternatively, they may repeat the whole process of parts (a) and (b) again on y – score for a full process leading to the solution for y in terms of t only.
A1: Correct simplified equation though may be with expanded brackets but like terms should be gathered (ie the $A\cos t$ and $B\sin t$ terms). If starting over, allow if names of constants are the same as for (b) (ie treat independently) but they will lose later marks. Accept complex index form.
(d)
M1: Realises the need to use the initial conditions to establish the values of their constants. But must have consistent constants (ie four different constants if they did part (c) from scratch). Solutions with complex indices are unlikely to achieve this mark. If unsure, use the Review system to consult your team leader.
dM1: Sets $x = y$ and collects and reaches $\tan t = \ldots$. Alternatively may convert into sine or cosine and reach an equivalent equation.
A1: Correct equation – accept equivalent if other trig approaches are used. Must come from correct work. Note $\sin t = \frac{7}{\sqrt{65}}$ and $\cos t = \frac{4}{\sqrt{65}}$ are the other ratios.
M1: Solves their $\tan t = \ldots$ and multiplies by 24. Must be working in radians. If degrees mode is used score M0.
A1: Correct value. Ignore units.
(e)
B1: **Must have at least one correct general equation for x or y for this mark to be awarded.** Rote answers about generic models without evidence to support any claims should not be given credit.
Suggests a suitable correct limitation of the model. This **must** focus on the fact the equations produce negative values. Award if a correct answer is given and ignore extra comments.
E.g. "gives negative values" would be a minimal acceptable answer.

Note: Answers about unlimited growth that imply the populations increase indefinitely or about scarcity of resource **without** an accompanying correct statement are B0.