

3.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

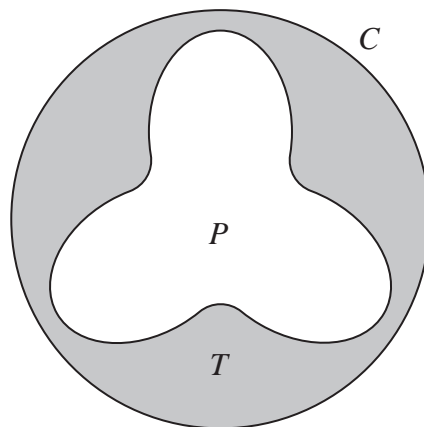


Figure 1

Figure 1 shows the design for a bathing pool.

The pool, P , shown unshaded in Figure 1, is surrounded by a tiled area, T , shown shaded in Figure 1.

The tiled area is bounded by the edge of the pool and by a circle, C , with radius 6 m.

The centre of the pool and the centre of the circle are the same point.

The edge of the pool is modelled by the curve with polar equation

$$r = 4 - a \sin 3\theta \quad 0 \leq \theta \leq 2\pi$$

where a is a positive constant.

Given that the shortest distance between the edge of the pool and the circle C is 0.5 m,

(a) determine the value of a .

(2)

(b) Hence, using algebraic integration, determine, according to the model, the exact area of T .

(6)