

4. The complex number $z = e^{i\theta}$, where θ is real.

(a) Show that

$$z^n + \frac{1}{z^n} \equiv 2 \cos n\theta$$

where n is a positive integer.

(2)

(b) Show that

$$\cos^5 \theta = \frac{1}{16} (\cos 5\theta + 5 \cos 3\theta + 10 \cos \theta)$$

(5)

(c) Hence, making your reasoning clear, determine all the solutions of

$$\cos 5\theta + 5 \cos 3\theta + 12 \cos \theta = 0$$

in the interval $0 \leq \theta < 2\pi$

(3)