5. A raindrop falls from rest from a cloud. The velocity, $v \, \text{m s}^{-1}$ vertically downwards, of the raindrop, t seconds after the raindrop starts to fall, is modelled by the differential equation

$$(t + 2)\frac{dv}{dt} + 3v = k(t + 2) - 3 \qquad t \geqslant 0$$

where k is a positive constant.

(a) Solve the differential equation to show that

$$v = \frac{k}{4}(t + 2) - 1 + \frac{4(2 - k)}{(t + 2)^3}$$

(5)

Given that $v = 4$ when $t = 2$

(b) determine, according to the model, the velocity of the raindrop 5 seconds after it starts to fall.

(3)

(c) Comment on the validity of the model for very large values of t

(1)