

8. A scientist is studying the effect of introducing a population of type  $A$  bacteria into a population of type  $B$  bacteria.

At time  $t$  days, the number of type  $A$  bacteria,  $x$ , and the number of type  $B$  bacteria,  $y$ , are modelled by the differential equations

$$\frac{dx}{dt} = x + y$$

$$\frac{dy}{dt} = 3y - 2x$$

- (a) Show that

$$\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 5x = 0 \tag{3}$$

- (b) Determine a general solution for the number of type  $A$  bacteria at time  $t$  days. (4)

- (c) Determine a general solution for the number of type  $B$  bacteria at time  $t$  days. (2)

The model predicts that, at time  $T$  hours, the number of bacteria in the two populations will be equal.

Given that  $x = 100$  and  $y = 275$  when  $t = 0$

- (d) determine the value of  $T$ , giving your answer to 2 decimal places. (5)

- (e) Suggest a limitation of the model. (1)