

Question	Scheme	Marks	AOs	
3(i)(a)	$ \mathbf{M} = 2(1+2) - a(-1-1) + 4(2-1) = 0 \Rightarrow a = \dots$	M1	2.3	
	The matrix \mathbf{M} has an inverse when $a \neq -5$	A1	1.1b	
		(2)		
(b)	Minors : $\begin{pmatrix} 3 & -2 & 1 \\ -a-8 & 2 & a+4 \\ 4-a & -6 & -2-a \end{pmatrix}$	B1	1.1b	
	or			
	Cofactors : $\begin{pmatrix} 3 & 2 & 1 \\ a+8 & 2 & -a-4 \\ 4-a & 6 & -2-a \end{pmatrix}$			
	$\mathbf{M}^{-1} = \frac{1}{ \mathbf{M} } \text{adj}(\mathbf{M})$	M1	1.1b	
	$\mathbf{M}^{-1} = \frac{1}{2a+10} \begin{pmatrix} 3 & a+8 & 4-a \\ 2 & 2 & 6 \\ 1 & -a-4 & -2-a \end{pmatrix}$	2 correct rows or columns. Follow through their det \mathbf{M}	A1ft	1.1b
		All correct. Follow through their det \mathbf{M}	A1ft	1.1b
	(4)			
(ii)	When $n = 1$, lhs = $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$, rhs = $\begin{pmatrix} 3^1 & 0 \\ 3(3^1-1) & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$	B1	2.2a	
	So the statement is true for $n = 1$			
	Assume true for $n = k$ so $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^k = \begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}$	M1	2.4	
	$\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$	M1	2.1	
	$= \begin{pmatrix} 3 \times 3^k & 0 \\ 3 \times 3(3^k-1) + 6 & 1 \end{pmatrix}$	A1	1.1b	
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 3(3^{k+1}-1) & 1 \end{pmatrix}$	A1	1.1b	
	If the statement is true for $n = k$ then it has been shown true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all positive integers n	A1	2.4	
	(6)			
(12 marks)				

Question 3 notes:**(i)(a)**

M1: Attempts determinant, equates to zero and attempts to solve for a in order to establish the restriction for a

A1: Provides the correct condition for a if **M** has an inverse

(i)(b)

B1: A correct matrix of minors or cofactors

M1: For a complete method for the inverse

A1ft: Two correct rows following through their determinant

A1ft: Fully correct inverse following through their determinant

(ii)

B1: Shows the statement is true for $n = 1$

M1: Assumes the statement is true for $n = k$

M1: Attempts to multiply the correct matrices

A1: Correct matrix in terms of k

A1: Correct matrix in terms of $k + 1$

A1: Correct complete conclusion