Question	Scheme	Marks	AOs
2(a)	A correct method to sum the series, most likely by the method of differences. Look for $\frac{10}{r^2+8r+15}=\frac{A}{r+3}+\frac{B}{r+5}\Rightarrow A=..,B=..$ followed by an attempt at the sum (or with 1 instead of 10). (Induction may be attempted – see alt for (a).)	M1	3.1a
	$\frac{10}{r^2+8r+15}=\frac{5}{r+3}-\frac{5}{r+5}$ or $\frac{1}{r^2+8r+15}=\frac{1/2}{r+3}-\frac{1/2}{r+5}$	B1	1.1b
	$\sum_{r=1}^{n}\frac{10}{r^2+8r+15}=5\sum_{r=1}^{n}\left(\frac{1}{r+3}-\frac{1}{r+5}\right)$ $=5\left[\left(\frac{1}{4}-\cancel{\frac{1}{6}}\right)+\left(\frac{1}{5}-\cancel{\frac{1}{7}}\right)+\left(\cancel{\frac{1}{6}}-\cancel{\frac{1}{8}}\right)+\cdots+\left(\cancel{\frac{1}{n+3}}-\frac{1}{n+5}\right)\right]$	M1	2.1
	$=5\left(\frac{1}{4}+\frac{1}{5}-\frac{1}{n+4}-\frac{1}{n+5}\right)$	A1ft	1.1b
	$=5\left(\frac{5(n+4)(n+5)+4(n+4)(n+5)-20(n+5)-20(n+4)}{20(n+4)(n+5)}\right)=\ldots$	M1	2.1
	$=\frac{9n^2+41n}{4(n+4)(n+5)}$ (So k = 4)	A1	1.1b
		(6)	
(b)	As $n\to\infty$, $T_n\to\frac{9}{4}$ or appropriate investigation tried.	M1	3.4
	Since the sum is increasing towards $\frac{9}{4}$ which is strictly less than 2.5 T_n can never reach 2.5, so the 2.5 million remaining tonnes of coal will not all be mined no matter how long the company keeps mining.	A1	3.2b
		(2)	
(c)	In the first 20 years $T_{20}=\frac{221}{120}$ million tonnes of coal have been mined, so $2.5-\frac{221}{120}=\frac{79}{120}$ tonnes remain.	M1	2.2b
	Hence $\frac{79}{120\times 20}$ extra tonnes per year need mining, so the new model is $M_r=\frac{79}{2400}+\frac{10}{r^2+8r+15}$.	A1ft	3.5c
		(2)	
		(10 marks)	

Notes:

(a)

M1: Attempts the sum using an appropriate method – ie method of differences. An attempt at partial fractions would evidence the attempt.

B1: Correct split into partial fractions.

M1: Applies method of differences showing evidence of the cancelling terms. The 5 may be missing at this stage and included later.

A1ft: Correct non-cancelling terms identified. Follow through their split into partial fractions if it leads to most terms cancelling.

M1: Puts the terms over a common denominator and simplifies. May be done in stages with the numerical fractions combined first etc, but look for appropriately adapted numerators for their method.

A1: Correct form with $k = 4$.

(b)

M1: Investigates the long term behaviour, e.g. by trying large values of n in the expression to see what happens, or by considering the long term limit.

A1: As scheme, comments that since the limit of the sum as $n \to \infty$ is 9/4 then the total amount of coal mined will never exceed 2.25 million tonnes, and so the coal will not all be mined even after a long time.

(c)

M1: Calculates the shortfall between 2.5 and the value of the sum at $n = 20$.

A1ft: Correct adaptation of the model adding (their shortfall)/20 to the original expression.

Alt (a)	Use of induction: Look for an attempt to find the value of k using $n = 1$ followed by an attempt at the inductive hypothesis.	M1	3.1a
	$n=1 \Rightarrow \dfrac{10}{1+8+15} = \dfrac{9+41}{k(5)(6)} \Rightarrow k=4$	B1	1.1b
	Assume true for $n = p$, so $\sum_{r=1}^{p+1} M_r = \dfrac{9p^2+41p}{"4"(p+4)(p+5)} + \dfrac{10}{(p+1)^2+8(p+1)+15}$ $= \dfrac{9p^2+41p}{"4"(p+4)(p+5)} + \dfrac{10}{(p+4)(p+6)}$	M1	2.1
	$= \dfrac{(9p^2+41p)(p+6)+10\times"4"(p+5)}{"4"(p+4)(p+5)(p+6)}$	M1 A1ft	1.1b 1.1b
	$= \dfrac{9p^3+95p^2+286p+200}{4(p+4)(p+5)(p+6)} = \dfrac{(p+4)(p+1)(9p+50)}{4(p+4)(p+5)(p+6)}$ $= \dfrac{(p+1)\left[9(p+1)^2+41\right]}{4((p+1)+4)((p+1)+5)}$ Hence true for $n = 1$ (with $k = 4$) and if true for $n = p$ then true for $n = p+1$ so true for all positive integers n.	A1	2.1
		(6)	

M1: For use of induction look for an attempt to find the value of k first, followed by an attempt at proving the inductive step.

B1: Deduces $k = 4$.

M1: Assumes true for some p and uses their k in the expression for T_p (may use k instead of p, which is fine if there is no confusion as they have a value in the expression).

M1: Attempts to combine over a common denominator.

A1ft: Correct single fraction expression, follow through their k.

A1: Completes the induction step and make a suitable conclusion.