Question	Scheme	Marks	AOs
4(a)	$\lvert w-2\rvert^2 = (w-2)(w-2)^* = (w-2)(w^*-2)$	M1	1.1b
	$= ww^* - 2w - 2w^* + 4 = \lvert w\rvert^2 - 2(w + w^*) + 4$	M1	1.1b
	$= 1 + 4 - 2(w + w^*) = 5 - 2(w + w^*)$ since w is a root of unity so has modulus 1. *	A1*	2.1
		(3)	
Alt	$w = x + \mathrm{i}y \Rightarrow \lvert w-2\rvert^2 = \lvert (x-2) + \mathrm{i}y\rvert^2 = (x-2)^2 + y^2$	M1	1.1b
	$= x^2 - 4x + 4 + y^2 = x^2 + y^2 + 4 - 2(x + \mathrm{i}y + x - \mathrm{i}y)$	M1	1.1b
	$=1+4+2(w+w^*)$ since $x^2 + y^2 = 1$ as w is a root of unity. *	A1*	2.1
		(3)	
(b)	$\sum_{i=1}^{7}(XA_i)^2 = \sum_{i=1}^{7}\lvert w_i - 2\rvert^2$ where w_i are the 7th roots of unity.	M1	3.1a
	$= \sum_{i=1}^{7}\left(5 - 2(w_i + w_i^*)\right) = \sum_{i=1}^{7} 5 - 2\sum_{i=1}^{7}(w_i + w_i^*)$	M1	1.1b
	$\sum_{i=1}^{7}(w_i + w_i^*) = 0$ since roots of unity sum to zero.	B1	2.2a
	So $\sum_{i=1}^{7}(XA_i)^2 = 7 \times 5 = 35$	A1	1.1b
		(4)	
			(7 marks)

Notes:

(a)

M1: Uses the given identity and distributivity of the conjugate.

M1: Expands and collects terms

A1*: Completes the proof with justification of $\lvert w\rvert = 1$.

Alt

M1: Replaces w by $x + \mathrm{i}y$ and applied the modulus squared.

M1: Expands the brackets and gathers $x^2 + y^2$ (may be implied if $x^2 + y^2 = 1$ stated explicitly) and splits the x term (may be implied if $w + w^* = 2x$ stated explicitly).

A1*: Completes proof convincingly with justification for $x^2 + y^2 = 1$ given.

(b)

M1: Makes the connection with part (a) and translates into a complex plane problem, realising the vertices lie at 7th roots of unity.

M1: Uses the identity shown in (a) and splits the sum.

B1: Deduces the second sum is zero as sum of roots of unity is zero.

A1: Correct answer.