Question	Scheme	Marks	AOs
5(a)	$\frac{dy}{dx}=\frac{1}{\sinh^2 x+1}\times..$	M1	1.2
	$\frac{dy}{dx}=\frac{1}{\sinh^2 x+1}\times\cosh x$	A1	1.1b
	$=\frac{\cosh x}{\cosh^2 x}=\text{sech}\, x$ or use of correct identity $\sinh^2 x + 1 = \cosh^2 x$ later in the proof.	B1	2.1
	E.g. $\frac{d^2y}{dx^2}=-\text{sech}\, x\tanh x$ or $\frac{d^2y}{dx^2}=-(\cosh x)^{-2}\times\sinh x$ or even $\frac{d^2y}{dx^2}=\frac{(\sinh x)(\sinh^2 x+1)-(\cosh x)(2\sinh x\cosh x)}{(\sinh^2 x+1)^2}$	M1	1.1b
	$\frac{d^3y}{dx^3}=-(-\text{sech}\, x\tanh x)(\tanh x)+(-\text{sech}\, x)(\text{sech}^2 x)$ (oe) or any valid attempt at the third derivative from their second derivative. E.g. $\frac{d^2y}{dx^2}=-\tanh x\frac{dy}{dx}$ then $\frac{d^3y}{dx^3}=-\text{sech}^2 x\frac{dy}{dx}-\tanh x\frac{d^2y}{dx^2}$	M1 A1	3.1a 1.1b
	E.g. $\frac{d^3y}{dx^3}=\text{sech}\, x\tanh^2 x-\text{sech}^3 x=\text{sech}\, x(1-\text{sech}^2 x)-\text{sech}^3 x$ $=\text{sech}\, x-2\,\text{sech}^3 x=\frac{dy}{dx}-2\left(\frac{dy}{dx}\right)^3$ * **or** $\frac{d^3y}{dx^3}=-\text{sech}^2 x\frac{dy}{dx}-\tanh x\frac{d^2y}{dx^2}=-\left(\frac{dy}{dx}\right)^3+\tanh^2 x\frac{dy}{dx}$ $=(1-\text{sech}^2 x)\frac{dy}{dx}-\left(\frac{dy}{dx}\right)^3=\frac{dy}{dx}-2\left(\frac{dy}{dx}\right)^3$ *	A1*	2.1
		(7)	
(b)	$\frac{d^4y}{dx^4}=\frac{d^2y}{dx^2}-6\left(\frac{dy}{dx}\right)^2\times\frac{d^2y}{dx^2}$	M1 A1	1.1b 1.1b
	$\frac{d^5y}{dx^5}=\frac{d^3y}{dx^3}-12\left(\frac{dy}{dx}\right)\times\left(\frac{d^2y}{dx^2}\right)^2-6\left(\frac{dy}{dx}\right)^2\frac{d^3y}{dx^3}$	M1 A1	2.1 1.1b
		(4)	
(c)	At $x=0, y=0, y'=1, y''=0, y^{(3)}=-1, y^{(4)}=0$ and $y^{(5)}=-1-1\times0^2-6\times1^2\times(-1)=5$	M1	1.1b
	So $y=y(0)+xy'(0)+\frac{x^2}{2!}y''(0)+\frac{x^3}{3!}y^{(3)}(0)+\frac{x^4}{4!}y^{(4)}(0)+\frac{x^5}{5!}y^{(5)}(0)+...$ with their evaluated values.	M1	1.1b
	$y=x-\frac{x^3}{6}+\frac{x^5}{24}+...$	A1	2.5
		(3)	
		(14 marks)	

Notes:

(a)

M1: Applies correct derivative of arctan(..)

A1: Correct derivative of *y*.

B1: Uses the identity $1+\sinh^2 x = \cosh^2 x$ to simplify the expression or anywhere later in their proof.

M1: Attempts the second derivative either using standard results, or quotient rule on unsimplified form.

M1: Simplifies and attempts the third derivative or attempts third derivative before simplifying. May even replace sech *x* with *y*' in the second derivative before using product rule. Many routes are possible at this stage (but must use product rule, chain rule, quotient rule as appropriate)

A1: A correct third derivative in any form.

A1*: Fully correct work leading to the given answer. Steps should be clear to reach the given answer.

(b)

M1: Differentiates again using the chain rule on the cube term. Constant multiple may be incorrect.

A1: Correct (unsimplified) fourth derivative.

M1: Completes the process of differentiation to reach the 5^{th} derivative.

A1: Correct answer, need not be simplified. Isw after a correct expression.

(c)

M1: Attempts the evaluation of all the derivatives at $x = 0$.

M1: Applies the Maclaurin formula with their values. Accept with 3! or 6 and with 5! or 120.

A1: Correct series, must start $y = ...$ or with $f(x) = ...$ only if this has been defined as being equal to *y* at some stage in their working.