Question	Scheme	Marks	AOs
6(a)	$\frac{d^2x}{dt^2}+6\frac{dx}{dt}+9x=2e^{-3t}$		
	AE: $m^2+6m+9=0 \Rightarrow (m+3)^2=0 \Rightarrow m=\ldots\ (=-3)$	M1	1.1b
	So C.F. is $x_{CF}=(A+Bt)e^{-3t}$	A1	2.2a
	For P.I. try $x_{PI}=kt^2e^{-3t}$	B1	2.2a
	$\dot{x}_{PI}=2kte^{-3t}-\ 3kt^2e^{-3t}\quad (=k(2t-3t^2)e^{-3t})$ $\ddot{x}_{PI}=2ke^{-3t}-6kte^{-3t}-6kte^{-3t}+9kt^2e^{-3t}\quad (=k(2-12t+9t^2)e^{-3t})$ $\Rightarrow\ k(2-12t+9t^2)e^{-3t}+6k(2t-3t^2)e^{-3t}+9kt^2e^{-3t}=2e^{-3t}\Rightarrow k=\ldots$	M1	1.1b
	So $k=1$ ie $x_{PI}=t^2e^{-3t}$	A1	1.1b
	General solution is $x=(A+Bt)e^{-3t}+t^2e^{-3t}$ (their C.F. + their P.I.)	M1	1.1a
	$x(0)=20 \Rightarrow A=20$	M1	3.4
	$\dot{x}=Be^{-3t}-3(A+Bt)e^{-3t}+2te^{-3t}-3t^2e^{-3t}=(B-3A+(2-3B)t-3t^2)e^{-3t}$ $\dot{x}(0)=100\Rightarrow B=100+3A=\ldots(=160)$	M1	3.4
	So $x=(20+160t+t^2)e^{-3t}$	A1	1.1b
		(9)	
(b)	From above $\dot{x}=(B-3A+(2-3B)t-3t^2)e^{-3t}=(100-478t-3t^2)e^{-3t}$		
	$\dot{x}=0\Rightarrow 100-478t-3t^2=0\Rightarrow t=..\quad (=-159.5\ldots \text{ or } 0.2089\ldots)$	M1	3.1a
	$t>0$, so $t_{max}=0.2089\ldots\Rightarrow$ $x_{max}=\left(20+160\times0.2089\ldots+(0.2089\ldots)^2\right)e^{-3\times0.2089\ldots}=\ldots$	M1	3.4
	$x_{max}=$ awrt 28.6 cm (3 s.f.) (28.57055381741878)	A1	1.1b
		(3)	
(c)	$x(2.86)=0.0912\ldots$ which is close to zero (less than 1mm), which can be accounted for by inaccuracies in measurements. So the model is supported by this measurement.	B1ft	2.2b
		(1)	

(13 marks)

Notes:

(a)

M1: Forms and solves the auxiliary equation.

A1: Deduces correct C.F. for repeated root. (Variables must be consistent.)

B1: Deduces a correct form for the P.I. following a correct C.F. Accept any variations that include kt^2e^{-3t} with other terms.

M1: Differentiates their P.I. twice and substitutes into original equation and attempts to find the unknown(s).

A1: Correct value for k or correct P.I.

M1: Forms general solution, x = their C.F. + their P.I.

M1: Uses $x=20$ at $t=0$ to find first constant/set up one equation in two unknowns.

M1: Differentiates general solution and uses $\dot{x}=100$ at $t=0$ to form and solve second equation in the unknowns.

A1: Correct answer.

(b) **M1:** Uses $\dot{x} = 0$ to find the time the maximum is achieved. May use the derivative from (a) with constants found, or may differentiate again from answer to (a). **M1:** Substitutes t_{max} into their equations to find x_{max}. **A1:** Correct answer.
(c) **B1ft:** Finds x when t = 2.86 and makes an inference about whether it supports the model or not. The conclusion should be relevant for their found value, if close to zero then should conclude in accordance with model as may have slight variance due to measurements not being accurate, if not close to zero, then should conclude that even taking inaccuracies into account the measurement does not fit with the model.