Question	Scheme	Marks	AOs
7(a)	$\int_k^8 \left((4k^2-1)y-(32k^2-k)\right)\,\mathrm{d}y = \left[(4k^2-1)\dfrac{y^2}{2}-(32k^2-k)y\right]_k^8$ $=(4k^2-1)\dfrac{8^2-k^2}{2}-(32k^2-k)(8-k)$	M1	1.1b
	$=\dfrac{1}{2}(4k^2-1)(8-k)(8+k)-(32k^2-k)(8-k)$ $=\dfrac{1}{2}(8-k)\left((4k^2-1)(8+k)-2(32k^2-k)\right)$	M1	3.1a
	$=\dfrac{1}{2}(8-k)\left(32k^2+4k^3-8-k-64k^2+2k\right)$ $=\dfrac{1}{2}(8-k)\left(4k^3-32k^2+k-8\right)*$	A1*	2.1
		(3)	
(b)	Uses $(\pi)\int x^2\,\mathrm{d}y$ with both of the curves and adds the results (a complete method to find the volume of the main body piece).	B1	3.1a
	Attempts $\int x^2\mathrm{d}y = \int \dfrac{y^6}{k^4}\,\mathrm{d}y = ..$	M1	1.1b
	So $(\pi)\int_0^k x^2\,\mathrm{d}y = \dfrac{(\pi)k^3}{7}$	A1	2.2a
	Attempts second curve $\int x^2\mathrm{d}y = \int \dfrac{(4k^2-1)y-(32k^2-k)}{-(32-4k)}\,\mathrm{d}y = ..$	M1	1.1b
	$(\pi)\int_k^8 x^2\mathrm{d}y = \dfrac{\frac{1}{2}(8-k)\left(4k^3-32k^2+k-8\right)}{-4(8-k)} = (\pi)\dfrac{1}{8}\left(8-k+32k^2-4k^3\right)$	M1	2.1
	So volume of body $=(\pi)\left(\dfrac{k^3}{7}+\dfrac{1}{8}\left(8-k+32k^2-4k^3\right)\right)$ Or total volume $=(\pi)\left(1+\dfrac{k^3}{7}+\dfrac{1}{8}\left(8-k+32k^2-4k^3\right)\right)$	A1	1.1b
	$\dfrac{\mathrm{d}V}{\mathrm{d}k}=0 \Rightarrow \dfrac{3k^2}{7}+\dfrac{1}{8}\left(1+64k-12k^2\right)=0$	M1	3.1a
	$\Rightarrow 60k^2-448k+7=0 \Rightarrow k=...$	M1	1.1b
	But $k > ½$ so must be k = awrt 7.45 cm	A1	3.2a
		(9)	

(c)	Volume of handle is $\pi r^2 h$ $(= \pi(0.5)^2 \times 4\ ) = \pi$	B1	2.2a
	So volume of spinning top is $V = \pi\left(1 + \dfrac{(7.45)^3}{7} + \dfrac{1}{8}\left(8-(7.45)+32(7.45)^2-4(7.45)^3\right)\right) = \ldots$	M1	1.1b
	= awrt 237 cm^3 (3 s.f.)	A1	1.1b
		(3)	
			(15 marks)

Notes:

(a)

M1: Correct attempt at integration and applies the limits.

M1: Applies completion of the square or expanding and factorising to obtain the factor $(k - 8)$ and removes this factor.

A1*: Correct completion, expands and collects terms inside the bracket. No errors seen and sufficient steps must be shown.

(b)

B1: Realises the needed to find the volume and attempts the formula at for both curves, adding the result. Note the π is not necessary at all for part (b).

M1: Attempts to make x^2 the subject of the first equation and attempts to integrate it. Power of k may be incorrect.

A1: Correct limits 0 and k applied to deduce the volume in terms of k for this section.

M1: Attempts the integral for top portion of body, make x^2 the subject, including the $32 - 4k$ in the denominator.

M1: Obtains the result using (a) and cancels the $(8 - k)$ term to achieve a cubic in k.

A1: Adds the results of the integrals to give the volume for the whole spinning top, or just the body. If the volume of the cylinder is incorrect, ignore this term for the accuracy – the non-constant terms should all be correct.

M1: Realises the need to differentiate the result and set equal to 0 to obtain the x value at any stationary points. Must be a valid attempt at differentiating.

M1: Solves the quadratic (usual rules).

A1: Correct answer, with second root rejected, or comment why this root gives the maximum.

(c) Allow marks for part (c) for the volume if the work is done in part (b).

B1: Deduces correct volume π for the handle. Cylinder formula or use of integration may be used. Award wherever seen - could be in (b).

M1: Substitutes their value for k into their volume formula. Dependent on the volume formula having been the sum of the three sections and including the factor π - handle must be included. May have been done in (b).

A1: Awrt 237cm^3 NB if this is given in (b) allow this mark unless a different answer is given in (c), in which case count the answer given in (c) as their answer.