

2. A company operating a coal mine is concerned about the mine running out of coal. It is estimated that 2.5 million tonnes of coal are left in the mine. The company wishes to mine all of this coal in 20 years.

In order to mine the coal in a regulated manner, the company models the amount of coal to be mined in the coming years by the formula

$$M_r = \frac{10}{r^2 + 8r + 15}$$

where  $M_r$  is the amount of coal, in millions of tonnes, mined in year  $r$ , with the first year being year 1

- (a) Show that, according to the model, the total amount of coal, in millions of tonnes, mined in the first  $n$  years is given by

$$T_n = \frac{9n^2 + 41n}{k(n+4)(n+5)}$$

where  $k$  is a constant to be determined.

(6)

- (b) Explain why, according to this model, the mine will never run out of coal.

(2)

The company decides to mine an extra fixed amount each year so that all the coal will be mined in exactly 20 years.

- (c) Refine the formula for  $M_r$  so that 2.5 million tonnes of coal will be exhausted in exactly 20 years of mining.

(2)