4. (a) Using the identity $zz^* = |z|^2$, or otherwise, show that if w is any root of unity then

$$|w - 2|^2 = 5 - 2(w + w^*)$$

(3)

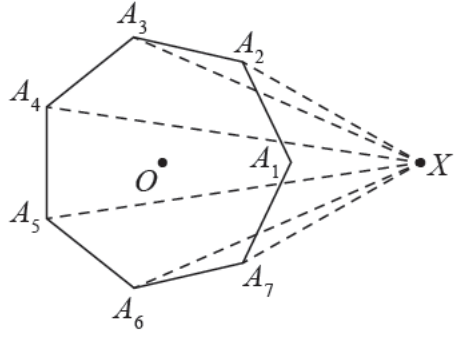

Figure 1

Figure 1 shows a regular heptagon $A_1A_2A_3A_4A_5A_6A_7$ whose vertices all lie on the unit circle with centre at the origin O and A_1 at (1, 0). The point X lies in the same plane as the heptagon and has coordinates (2, 0).

Using the result given in part (a),

(b) find $\sum_{i=1}^{7} \left(XA_i\right)^2$

(4)