

6. A damped spring is part of a car suspension system. In tests for the system, a mass is attached to the damped spring and is made to move upwards in a vertical line.

The motion of the system is modelled by the differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 2e^{-3t}$$

where x cm is the vertical displacement of the mass above its equilibrium position and t is the time, in seconds, after motion begins.

In one particular test, the mass is moved to a position 20 cm above its equilibrium position and given an initial velocity of 1 ms^{-1} upwards. For this test, use the model to

- (a) find an equation for x in terms of t , (9)

- (b) find, to the nearest mm, the maximum displacement of the mass from its equilibrium position. (3)

In this test, the time taken for the mass to return to its equilibrium position was measured as 2.86 seconds.

- (c) State, with justification, whether or not this supports the model. (1)