Question	Scheme	Marks	AOs
1(a)	$y=\tanh^{-1}(x)\Rightarrow \tanh y = x \Rightarrow x=\dfrac{\sinh y}{\cosh y}=\dfrac{e^{y}-e^{-y}}{e^{y}+e^{-y}}$	M1 A1	2.1 1.1b
	Note that some candidates only have one variable and reach e.g. $x=\dfrac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$ or $\tanh x=\dfrac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$ Allow this to score M1A1		
	$x\left(e^{2y}+1\right)=e^{2y}-1\Rightarrow e^{2y}(1-x)=1+x\Rightarrow e^{2y}=\dfrac{1+x}{1-x}$	M1	1.1b
	$e^{2y}=\dfrac{1+x}{1-x}\Rightarrow 2y=\ln\left(\dfrac{1+x}{1-x}\right)\Rightarrow y=\dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$ *	A1*	2.1
	Note that $e^{2y}(x-1)+x+1=0$ can be solved as a quadratic in e^{y}: $e^{y}=\dfrac{-\sqrt{0-4(x-1)(x+1)}}{2(x-1)}=\dfrac{-\sqrt{4(1-x)(x+1)}}{2(x-1)}=\dfrac{2\sqrt{(1-x)(x+1)}}{2(1-x)}$ $=\dfrac{\sqrt{(x+1)}}{\sqrt{(1-x)}}\Rightarrow y=\dfrac{1}{2}\ln\dfrac{(x+1)}{(1-x)}$ * Score **M1** for an attempt at the quadratic formula to make e^{y} the subject (condone $\pm\sqrt{\ldots}$) and **A1*** for a correct solution that rejects the positive root at some point and deals with the $(x-1)$ bracket correctly		
	$k=1$ or $-1<x<1$	B1	1.1b
		(5)	
(a) **Way 2**	$\tanh^{-1}x=\dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)\Rightarrow x=\tanh\left(\dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)\right)=\dfrac{e^{\ln\frac{1+x}{1-x}}-1}{e^{\ln\frac{1+x}{1-x}}+1}$	M1 A1	2.1 1.1b
	$x=\dfrac{e^{\ln\frac{1+x}{1-x}}-1}{e^{\ln\frac{1+x}{1-x}}+1}=\dfrac{\frac{1+x}{1-x}-1}{\frac{1+x}{1-x}+1}=x$ Hence true, QED, tick etc.	M1 A1	1.1b 2.1
(b)	$2x=\tanh\left(\ln\sqrt{2-3x}\right)\Rightarrow \tanh^{-1}(2x)=\ln\sqrt{2-3x}$	M1	3.1a
	$\dfrac{1}{2}\ln\left(\dfrac{1+2x}{1-2x}\right)=\dfrac{1}{2}\ln(2-3x)\Rightarrow\dfrac{1+2x}{1-2x}=2-3x$	M1	2.1
	$6x^{2}-9x+1=0$	A1	1.1b
	$6x^{2}-9x+1=0\Rightarrow x=\ldots$	M1	1.1b
	$x=\dfrac{9-\sqrt{57}}{12}$	A1	3.2a
		(5)	

	Alternative for first 2 marks of (b)		
	$2x=\tanh\left(\ln\sqrt{2-3x}\right)\Rightarrow 2x=\dfrac{e^{2\ln\sqrt{2-3x}}-1}{e^{2\ln\sqrt{2-3x}}+1}$	M1	3.1a
	$\Rightarrow \dfrac{2-3x-1}{2-3x+1}=2x$	M1	2.1
			(10 marks)

Notes

(a)

<u>If you come across any attempts to use calculus to prove the result – send to review</u>

M1: Begins the proof by expressing tanh in terms of exponentials and forms an equation in exponentials.

The exponential form can be any of $\dfrac{\left(e^{y}-e^{-y}\right)/2}{\left(e^{y}+e^{-y}\right)/2}$, $\dfrac{e^{y}-e^{-y}}{e^{y}+e^{-y}}$, $\dfrac{e^{2y}-1}{e^{2y}+1}$

Allow any variables to be used **but the final answer must be in terms of *x***. Allow alternative notation for $\tanh^{-1}x$ e.g. artanh, arctanh.

A1: Correct expression for "x" in terms of exponentials

M1: Full method to make $e^{2\text{"}y\text{"}}$ the subject of the formula. This must be correct algebra so allow sign errors only.

A1*: Completes the proof by using logs correctly and reaches the printed answer with no errors.

Allow e.g. $\dfrac{1}{2}\ln\left(\dfrac{x+1}{1-x}\right)$, $\dfrac{1}{2}\ln\dfrac{x+1}{1-x}$, $\dfrac{1}{2}\ln\left|\dfrac{x+1}{1-x}\right|$. Need to see $\tanh^{-1}x=\dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$ as a conclusion

but allow if the proof concludes that $y=\dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$ with y defined as $\tanh^{-1}x$ earlier.

B1: Correct value for k or writes $-1<x<1$

Way 2

M1: Starts with result, takes tanh of both sides and expresses in terms of exponentials

A1: Correct expression

M1: Eliminates exponentials and logs and simplifies

A1: Correct result (i.e. $x = x$) with conclusion

B1: Correct value for k or writes $-1<x<1$

(b)

M1: Adopts a correct strategy by taking $\tanh^{-1}$ of both sides

M1: Makes the link with part (a) by replacing artanh($2x$) with $\dfrac{1}{2}\ln\left(\dfrac{1+2x}{1-2x}\right)$ and demonstrates the use of the power law of logs to obtain an equation with logs removed **correctly**.

A1: Obtains the correct 3TQ

M1: Solves their 3TQ using a correct method (see General Guidance – if no working is shown (calculator) and the roots are correct for their quadratic, allow M1)

A1: Correct value with the other solution rejected (accept rejection by omission) so $x=\dfrac{9\pm\sqrt{57}}{12}$ scores A0 unless the positive root is rejected

Alternative for first 2 marks of (b)

M1: Adopts a correct strategy by expressing tanh in terms of exponentials

M1: Demonstrates the use of the power law of logs to obtain an equation with logs removed correctly