Question	Scheme	Marks	AOs
3(a) Way 1	$x=\frac{3}{2}\sinh u$	B1	2.1
	$\int\frac{dx}{\sqrt{4x^2+9}}=\int\frac{1}{\sqrt{4\left(\frac{9}{4}\right)\sinh^2 u+9}}\times\frac{3}{2}\cosh u\ du$	M1	3.1a
	$=\int\frac{1}{2}\ du$	A1	1.1b
	$=\int\frac{1}{2}\ du=\frac{1}{2}u=\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)+c$	A1	1.1b
		(4)	
(a) Way 2	$x=\frac{3}{2}\tan u$	B1	2.1
	$\int\frac{dx}{\sqrt{4x^2+9}}=\int\frac{1}{\sqrt{4\left(\frac{9}{4}\right)\tan^2 u+9}}\times\frac{3}{2}\sec^2 u\ du$	M1	3.1a
	$=\int\frac{1}{2}\sec u\ du$	A1	1.1b
	$=\frac{1}{2}\ln\left(\sec u+\tan u\right)=\frac{1}{2}\ln\left(\frac{2x}{3}+\sqrt{1+\left(\frac{2x}{3}\right)^2}\right)$ $u=\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)+c$	A1	1.1b
(a) Way 3	$x=\frac{1}{2}u$ or $x=ku$ where $k>0\ \ k\neq 1$	B1	2.1
	$\int\frac{dx}{\sqrt{4x^2+9}}=\int\frac{1}{\sqrt{4\left(\frac{1}{4}\right)u^2+9}}\times\frac{1}{2}du$	M1	3.1a
	$=\frac{1}{2}\int\frac{1}{\sqrt{u^2+9}}du\ \left(\text{or }\frac{1}{2}\int\frac{1}{\sqrt{u^2+\frac{9}{4k^2}}}du\ \text{ for } x=ku\right)$	A1	1.1b
	$=\frac{1}{2}\sinh^{-1}\frac{u}{3}=\frac{1}{2}\sinh^{-1}\frac{2x}{3}+c$	A1	1.1b
(b)	Mean value = $\frac{1}{3(-0)}\left[\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)\right]_0^3=\frac{1}{3}\times\frac{1}{2}\sinh^{-1}\left(\frac{2\times 3}{3}\right)(-0)$	M1	2.1
	$=\frac{1}{6}\ln\left(2+\sqrt{5}\right)$ **(Brackets are required)**	A1ft	1.1b
		(2)	
			(6 marks)

Notes
(a) B1: Selects an appropriate substitution leading to an integrable form M1: Demonstrates a fully correct method for the substitution that includes substituting into the function and dealing with the "dx". The substitution being substituted does not need to be "correct" for this mark but the substitution must be an attempt at $\int \frac{1}{\sqrt{4\left[\mathrm{f}(u)\right]^2+9}} \times \mathrm{f}'(u)\,\mathrm{d}u$ with the $\mathrm{f}'(u)$ correct for their substitution. E.g. if $x = \frac{1}{2}u$ is used, must see dx = $\frac{1}{2}\mathrm{d}u$ not 2du. A1: Correct simplified integral in terms of u from correct work and from a correct substitution A1: Correct answer including "+ c". Allow arcsinh or arsinh for $\sinh^{-1}$ from correct work and from a correct substitution (b) M1: Correctly applies the method for the mean value for their integration which must be of the form specified in part (a) and substitutes the limits 0 and 3 but condone omission of 0 A1: Correct exact answer (follow through their A and B). **Brackets are required if appropriate.**