Question	Scheme	Marks	AOs
4(a) Way 1	$C+iS=\cos\theta+i\sin\theta+\frac{1}{2}(\cos5\theta+i\sin5\theta)\left(+\frac{1}{4}(\cos9\theta+i\sin9\theta)+...\right)$	M1	1.1b
	$=e^{i\theta}+\frac{1}{2}e^{5i\theta}\left(+\frac{1}{4}e^{9i\theta}+...\right)$	A1	2.1
	$C+iS=\frac{e^{i\theta}}{1-\frac{1}{2}e^{4i\theta}}$	M1	3.1a
	$=\frac{2e^{i\theta}}{2-e^{4i\theta}}*$	A1*	1.1b
		(4)	
(a) Way 2	$C+iS=\cos\theta+i\sin\theta+\frac{1}{2}(\cos5\theta+i\sin5\theta)\left(+\frac{1}{4}(\cos9\theta+i\sin9\theta)+...\right)$	M1	1.1b
	$C+iS=\cos\theta+i\sin\theta+\frac{1}{2}(\cos\theta+i\sin\theta)^5\left(+\frac{1}{4}(\cos\theta+i\sin\theta)^9+...\right)$	A1	2.1
	$C+iS=\frac{\cos\theta+i\sin\theta}{1-\frac{1}{2}(\cos\theta+i\sin\theta)^4}=\frac{e^{i\theta}}{1-\frac{1}{2}e^{4i\theta}}$	M1	3.1a
	$=\frac{2e^{i\theta}}{2-e^{4i\theta}}*$	A1*	1.1b
		(4)	
(b) Way 1	$\frac{2e^{i\theta}}{2-e^{4i\theta}}\times\frac{2-e^{-4i\theta}}{2-e^{-4i\theta}}$	M1	3.1a
	$\frac{4e^{i\theta}-2e^{-3i\theta}}{4-2e^{-4i\theta}-2e^{4i\theta}+1}$	A1	1.1b
	$\frac{4\cos\theta+4i\sin\theta-2\cos3\theta+2i\sin3\theta}{5-2\cos4\theta+2i\sin4\theta-2\cos4\theta-2i\sin4\theta}$ **Dependent on the first M**	**d**M1	2.1
	$S=\frac{4\sin\theta+2\sin3\theta}{5-4\cos4\theta}*$	A1*	1.1b
		(4)	
(b) Way 2	$\frac{2e^{i\theta}}{2-e^{4i\theta}}=\frac{2(\cos\theta+i\sin\theta)}{2-(\cos4\theta+i\sin4\theta)}\times\frac{2-(\cos4\theta-i\sin4\theta)}{2-(\cos4\theta-i\sin4\theta)}$	M1	3.1a
	$\frac{4\cos\theta+4i\sin\theta-2\cos\theta\cos4\theta-2\sin\theta\sin4\theta+2i\sin4\theta\cos\theta-2i\sin\theta\cos4\theta}{4+\cos^2 4\theta+\sin^2 4\theta-4\cos4\theta}$	A1	1.1b
	$\frac{4\cos\theta+4i\sin\theta-2\cos3\theta+2i\sin3\theta}{5-2\cos4\theta+2i\sin4\theta-2\cos4\theta-2i\sin4\theta}$ **Dependent on the first M**	**d**M1	2.1
	$S=\frac{4\sin\theta+2\sin3\theta}{5-4\cos4\theta}*$	A1*	1.1b

(8 marks)

Notes

(a)

Way 1

M1: Combines the two series by pairing the multiples of θ (At least up to 5θ)

A1: Converts to Euler form correctly (At least up to 5θ)

M1: Recognises that C + iS is a convergent geometric series and uses the sum to infinity of a GP

A1*: Reaches the printed answer with no errors

Way 2

M1: Combines the two series by pairing the multiples of θ (At least up to 5θ)

A1: Converts to power form correctly (At least up to 5θ)

M1: Recognises that C + iS is a convergent geometric series and uses the sum to infinity of a GP

A1*: Reaches the printed answer with no errors

(b)

Way 1

M1: Multiplies numerator and denominator by $2-e^{-4i\theta}$

A1: Correct fraction in terms of exponentials

dM1: Converts back to trigonometric form

A1*: Reaches the printed answer with no errors

Way 2

M1: Converts back to trigonometric form and realises the need to make the denominator real and multiplies numerator and denominator by the complex conjugate of the denominator which is **correct** for their fraction

A1: Correct fraction in terms of trigonometric functions

dM1: Uses the correct addition formula to obtain $\sin 3\theta$ in the numerator

A1*: Reaches the printed answer with no errors