| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 5(a) | $4 m^{2}+4 m+37=0 \Rightarrow m=-\frac{1}{2} \pm 3 \mathrm{i}$ | M1 | 1.1b |
|  | $h=\mathrm{e}^{-0.5 t}(A \cos 3 t+B \sin 3 t)$ | A1 | 1.1b |
|  |  | (2) |  |
| (b) | $t=0, h=-20 \Rightarrow A=-20$ | M1 | 3.4 |
|  | $\begin{gathered} \frac{\mathrm{d} h}{\mathrm{~d} t}=-0.5 \mathrm{e}^{-05 t}(A \cos 3 t+B \sin 3 t)+\mathrm{e}^{-05 t}(-3 A \sin 3 t+3 B \cos 3 t) \\ t=0, \frac{\mathrm{~d} h}{\mathrm{~d} t}=55 \Rightarrow B=\ldots(\mathrm{NB} B=15) \end{gathered}$ | M1 | 3.4 |
|  | $(h=) \mathrm{e}^{-0.5 t}(15 \sin 3 t-20 \cos 3 t)$ | A1 | 1.1b |
|  | $\begin{gathered} -0.5 \mathrm{e}^{-05 t}(15 \sin 3 t-20 \cos 3 t)+\mathrm{e}^{-05 t}(60 \sin 3 t+45 \cos 3 t)=0 \\ \text { or e.g. } \\ -0.5 \mathrm{e}^{-05 t}(15 \sin 3 t-20 \cos 3 t)+\frac{25 \sqrt{37}}{2} \mathrm{e}^{-05 t} \sin \left(3 t+\arctan \frac{22}{21}\right)=0 \\ \Rightarrow t=\ldots \end{gathered}$ | M1 | 3.16 |
|  | $\tan 3 t=-\frac{22}{21} \text { or e.g. } 3 t+\tan ^{-1} \frac{22}{21}=0$ | $\begin{gathered} \text { A1 } \\ \text { M1 } \\ \text { on } \\ \text { ePEN } \end{gathered}$ | 2.1 |
|  | $t=0.778 \mathrm{~s}$ | A1 | 1.1b |
|  | $h=\mathrm{e}^{-0.5 \times " 0.778 "}\left(15 \sin \left(3 \times\right.\right.$ "0.778") $\left.-20 \cos \left(3 \times 10.778^{\prime \prime}\right)\right)$ | dM1 | 1.1b |
|  | $=16.7 \mathrm{~cm}$ | A1 | 3.2a |
|  |  | (8) |  |
| (c) | E.g. considers large values of $t$ in the model for $h$ or states that for large values of $t, h$ becomes smaller or becomes zero | M1 | 3.4 |
|  | E.g. <br> - The value of $h$ is very small when $t$ is large and this is likely to be correct (as the displacement of end of the board should get smaller and smaller) <br> - This suggests the model is suitable <br> - This is realistic <br> - This is suitable as the board will tend towards its equilibrium position <br> - When $t$ is large the value of $h$ is never zero so the model is not really appropriate for large values of $t$ | A1 <br> B1 on <br> ePEN | 3.2b |
|  |  | (2) |  |

(12 marks)

## Notes

## (a)

M1: Uses the model to form and solve the auxiliary equation $4 m^{2}+4 m+37=0$
See General Guidance for awarding this mark. This can be implied by correct values for $m$ (from calculator)
A1: Correct general solution including " $h=$ "
(b)

M1: Uses the model and the initial conditions to establish the value of " $A$ ". Need to see $t=0$ and $h= \pm 20$ leading to a value for " $A$ ". This may be implied by $A=-20$ or $A=20$.
M1: Differentiates their model using the product rule and uses the initial conditions, $t=0$ with $\mathrm{d} h / \mathrm{d} t= \pm 55$, to establish the value of " $B$ "
A1: Correct particular solution or correct values for $A$ and $B$
M1: Uses their solution to the model with a correct strategy to obtain a value for $t$ e.g.
differentiates or uses their derivative from earlier, sets equal to zero and solves for $t$
A1(M1 on ePEN): Correct equation for $t$
A1: Correct value for $t$ (allow awrt 0.778 if necessary) but this value may be implied.
dM1: Uses the model and their positive value for $t$ to find the maximum displacement - if their $t$
is incorrect there must be some indication that they are using their $h$ and not just a number written down. E.g. must see substitution into their $h$ or they re-state their $h$ and obtain a value for $h$.
Dependent on all the previous method marks
A1: Correct value (awrt 16.7 (units not needed))
(c)

M1: Considers the model for large values of $t$ either by substituting values or by considering the expression and commenting on its behaviour for large values of $t$. E.g. as $t \rightarrow \infty, h \rightarrow 0$ or as
$t \rightarrow \infty, e^{-05 t} \rightarrow 0$ or as $t \rightarrow \infty$ the oscillations become smaller etc.
A1: Makes a suitable comment - see scheme for examples

