Question	Scheme	Marks	AOs
8(a)	$k = 2.6$	B1	3.4
		(1)	
(b)	$x = 1.18 \Rightarrow \ln\left(3.6\times 1.18 - "2.6"\right) = \ldots$	M1	1.1b
	$h = 0.4995\ldots$ m	A1	2.2b
		(2)	
(c)	$y = \ln\left(3.6x - 2.6\right) \Rightarrow x = \dfrac{\mathrm{e}^y + 2.6}{3.6}$ or $\dfrac{5\mathrm{e}^y + 13}{18}$	B1ft	1.1a
	$V = \pi \int \left(\dfrac{\mathrm{e}^y + 2.6}{3.6}\right)^2 \mathrm{d}y = \dfrac{\pi}{3.6^2} \int \left(\mathrm{e}^{2y} + 5.2\mathrm{e}^y + 6.76\right)\mathrm{d}y$ or $\dfrac{\pi}{324} \int \left(25\mathrm{e}^{2y} + 130\mathrm{e}^y + 169\right)\mathrm{d}y$	M1	3.3
	$= \dfrac{\pi}{3.6^2}\left[\dfrac{1}{2}\mathrm{e}^{2y} + 5.2\mathrm{e}^y + 6.76y\right]$ $\left(\text{or } \dfrac{\pi}{324}\left[\dfrac{25}{2}\mathrm{e}^{2y} + 130\mathrm{e}^y + 169y\right]\right)$	A1	1.1b
	$= \dfrac{\pi}{3.6^2}\left\{\left(\dfrac{1}{2}\mathrm{e}^{2h} + 5.2\mathrm{e}^h + 6.76h\right) - \left(\dfrac{1}{2}\mathrm{e}^0 + 5.2\mathrm{e}^0 + 6.76(0)\right)\right\}$ or e.g. $= \dfrac{\pi}{324}\left\{\left(\dfrac{25}{2}\mathrm{e}^{2h} + 130\mathrm{e}^h + 169h\right) - \left(\dfrac{25}{2}\mathrm{e}^0 + 130\mathrm{e}^0 + 6.76(0)\right)\right\}$	M1	2.1
	$= \dfrac{\pi}{3.6^2}\left(\dfrac{1}{2}\mathrm{e}^{2h} + 5.2\mathrm{e}^h + 6.76h - 5.7\right)$	A1	1.1b
		(5)	
(d)	$\dfrac{\mathrm{d}V}{\mathrm{d}h} = \dfrac{\pi}{3.6^2}\left(\mathrm{e}^{2h} + 5.2\mathrm{e}^h + 6.76\right) = \dfrac{\pi}{3.6^2}\left(\mathrm{e}^{0\,4} + 5.2\mathrm{e}^{0\,2} + 6.76\right)$	M1	3.1a
	$\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{\mathrm{d}h}{\mathrm{d}V}\dfrac{\mathrm{d}V}{\mathrm{d}t} = \dfrac{1}{3.539\ldots} \times 0.015 \times 60$	M1	1.1b
	$\dfrac{\mathrm{d}h}{\mathrm{d}t} = 25.4\,\mathrm{cm\,h^{-1}}$	A1	3.2a
		(3)	
(d) Way 2	$y = 0.2 \Rightarrow x = \dfrac{2.6 + \mathrm{e}^{0\,2}}{3.6} \Rightarrow A = \pi\left(\dfrac{2.6 + \mathrm{e}^{0\,2}}{3.6}\right)^2 \left(= 3.54\right)$	M1	3.1a
	$\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{0.015 \times 60}{3.54}$	M1	1.1b
	$\dfrac{\mathrm{d}h}{\mathrm{d}t} = 25.4\,\mathrm{cm\,h^{-1}}$	A1	3.2a
			(11 marks)

Notes

(a)
B1: Uses the model to obtain a correct value for *k*. Must be 2.6 not -2.6
(b)

M1: Substitutes their value of k and $x = 1.18$ into the given model to find a value for y
A1: Infers that the depth of the pool could be awrt 0.5 m
(c)
B1ft: Uses the model to obtain x correctly in terms of y (follow through their k)
M1: Uses the model to obtain an expression for the volume of the pool using $\pi \int (\textit{their } f(y))^2 \, \mathrm{d}y$ – must expand in order to reach an integrable form (allow poor squaring e.g. $(a + b)^2 = a^2 + b^2$. **Note that the π may be recovered later.**
A1: Correct integration
M1: Selects limits appropriate to the model (h and 0) substitutes and clearly shows the use of both limits (i.e. including zero)
A1: Correct expression **(allow unsimplified and isw if necessary)**
(d)
Way 1
M1: Recognises that $\frac{\mathrm{d}V}{\mathrm{d}h}$ is required and attempts to find $\frac{\mathrm{d}V}{\mathrm{d}h}$ or $\frac{\mathrm{d}h}{\mathrm{d}V}$ from their integration or using the earlier result (before integrating). Must clearly be identified as $\frac{\mathrm{d}V}{\mathrm{d}h}$ or $\frac{\mathrm{d}h}{\mathrm{d}V}$ unless this implied by subsequent work.
M1: Evidence of the correct use of the chain rule (ignore any confusion with units). Look for an attempt to divide 15 or their converted 15 by their $\frac{\mathrm{d}V}{\mathrm{d}h}$ or to multiply 15 or their converted 15 by $\frac{\mathrm{d}h}{\mathrm{d}V}$ **but must reach a <u>value</u> for $\frac{\mathrm{d}h}{\mathrm{d}t}$ <u>but you do not need to check their value.</u>**
A1: Interprets their solution correctly to obtain the correct answer (awrt 25.4) **with the correct units**
Way 2
M1: Uses $y = 0.2$ to find x and the surface area of the water at that instant
M1: Attempts to divide the rate by their area (ignore any confusion with units)
A1: Interprets their solution correctly to obtain the correct answer (awrt 25.4) **with the correct units**