

4. The infinite series C and S are defined by

$$C = \cos \theta + \frac{1}{2} \cos 5\theta + \frac{1}{4} \cos 9\theta + \frac{1}{8} \cos 13\theta + \dots$$

$$S = \sin \theta + \frac{1}{2} \sin 5\theta + \frac{1}{4} \sin 9\theta + \frac{1}{8} \sin 13\theta + \dots$$

Given that the series C and S are both convergent,

(a) show that

$$C + iS = \frac{2e^{i\theta}}{2 - e^{4i\theta}} \quad (4)$$

(b) Hence show that

$$S = \frac{4\sin \theta + 2\sin 3\theta}{5 - 4\cos 4\theta} \quad (4)$$