Question	Scheme	Marks	AOs
7 (a)	$\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$	B1	1.2
	$\dfrac{e^x - e^{-x}}{e^x + e^{-x}} = y \Rightarrow e^x - e^{-x} = y(e^x + e^{-x}) = ye^x + ye^{-x}$ $\Rightarrow e^x(1-y) = e^{-x}(1+y) \Rightarrow e^{2x} = \dfrac{1+y}{1-y}$ or $\dfrac{e^y - e^{-y}}{e^y + e^{-y}} = x \Rightarrow e^y - e^{-y} = x(e^y + e^{-y}) = xe^y + xe^{-y}$ $\Rightarrow e^y(1-x) = e^{-y}(1+x) \Rightarrow e^{2y} = \dfrac{1+x}{1-x}$	M1	2.1
	$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$*	A1*	1.1b
		(3)	
(b)	$x = \frac{1}{2}\ln\left(\frac{1+0.8}{1-0.8}\right) = \frac{1}{2}\ln(9) = \ln 3$	B1	3.1a
	$V = \pi\int_0^{\ln 3} \tanh^2 x \, dx$	B1	1.1b
	$= \{\pi\}\int_0^{\ln 3}(1 - \operatorname{sech}^2 x)\, dx = [x - \tanh x]_0^{\ln 3} = [\ln 3 - 0.8] - [0]$	M1	3.1a
	$= \pi[\ln 3 - 0.8]$	A1	1.1b
	Volume cylinder = $\pi \times 0.8^2 \times' \ln 3'$ or $= \pi\int_0^{\ln 3} 0.8^2 \, dx$	M1	1.1b
	Volume = cylinder – solid of revolution	M1	3.1a
	Volume = $\pi\left[\frac{4}{5} - \frac{9}{25}\ln 3\right]$	A1	1.1b
		(7)	
			(10 marks)

Notes:

(a)

B1: Correct expression for $\tanh x$

M1: Sets equal to y and rearranges to get $e^{2x} = f(y)$. Alternatively switches x and y and rearranges to get $e^{2y} = f(x)$.

A1*: Achieves the printed answer with no errors seen

(b)

B1: $x = \ln 3$

B1: Correct expression for the volume of the curve including π, dx can be implied later work and follow through upper limit

M1: Uses the identity $\tanh^2 x = 1 - \operatorname{sech}^2 x$ to complete the integration and correct use of limits

A1: Correct volume of the curve

M1: A correct method to find the volume of the required cylinder

M1: A correct method to find the required volume.

A1: Correct volume in the required form