Question	Scheme	Marks	AOs
8(a)	$\mathbf{MM}^{\mathrm{T}} = \begin{pmatrix} 1 & 4 & -1 \\ 3 & 0 & p \\ q & r & s \end{pmatrix} \begin{pmatrix} 1 & 3 & q \\ 4 & 0 & r \\ -1 & p & s \end{pmatrix} = \begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix} \Rightarrow 3 - p = 0$	M1	2.1
	$p = 3$ *	A1*	1.1b
		(2)	
(b)	$k = 18$	B1	2.2a
		(1)	
(c)	$\mathbf{M}^{-1} = \dfrac{1}{18} \begin{pmatrix} 1 & 3 & q \\ 4 & 0 & r \\ -1 & 3 & s \end{pmatrix}$	B1ft	2.2a
		(1)	
(d)	Finds any two equations involving q, r and s from $q + 4r - s = 0, \ 3q + 3s = 0, \ q^2 + r^2 + s^2 = 18$	M1	3.1a
	All three correct equations $q + 4r - s = 0, \ 3q + 3s = 0, \ q^2 + r^2 + s^2 = 18$	A1	1.1b
	$s = -q, r = -\dfrac{1}{2}q \ \Rightarrow q^2 + \left(-\dfrac{1}{2}q\right)^2 + (-q)^2 = 18 \ \Rightarrow q = 2\sqrt{2}$ or $q = -2r, s = 2r \Rightarrow (-2r)^2 + r^2 + (2r)^2 = 18 \ \Rightarrow r = -\sqrt{2}$ or $q = -s, r = \dfrac{1}{2}s \Rightarrow (-s)^2 + \left(\dfrac{1}{2}s\right)^2 + s^2 = 18 \ \Rightarrow s = -2\sqrt{2}$	M1 A1	3.1a 1.1b
	$q = 2\sqrt{2}, r = -\sqrt{2}$ and $s = -2\sqrt{2}$ only	M1 A1	1.1b 2.2a
		(6)	
		(10 marks)	

Notes:

(a)

M1: Sets $\mathbf{MM}^{\mathrm{T}} = k\mathbf{I}$ and finds a value for p

A1*: Correct value for p

(b)

B1: Correct value for k

(c)

B1ft: Deduces $\mathbf{M}^{-1}$, follow through on their value of k

(d)

M1: Uses $\mathbf{MM}^{\mathrm{T}} = k\mathbf{I}$ with $p = 3$ and their value of k to find at least two equations involving at least two of the constants q, r and s

A1: All three correct equations

M1: A complete method to solve the equations to find a value for either q, r or s

A1: A correct constant

M1: Finds the other two constants

A1: Deduces all three correct constants

Alternative method using $\begin{pmatrix} 1 & 3 & q \\ 4 & 0 & r \\ -1 & 3 & s \end{pmatrix}\begin{pmatrix} 1 & 4 & -1 \\ 3 & 0 & 3 \\ q & r & s \end{pmatrix} = \begin{pmatrix} 18 & 0 & 0 \\ 0 & 18 & 0 \\ 0 & 0 & 18 \end{pmatrix}$

M1: First row times first column $1 + 9 + q^2 = 18 \implies q = ...$

A1: Correct value for q, $q = 2\sqrt{2}$ $\quad (q > 0)$

M1: First row time second column $4 + qr = 0 \implies r = ...$

A1: Correct value for r, $r = -\dfrac{4}{2\sqrt{2}} = -\sqrt{2}$

M1: First row time third column $-1 + 9 + qs = 0 \implies s = ...$

A1: Deduces the correct value for s, $s = -\dfrac{8}{2\sqrt{2}} = -2\sqrt{2}$