

7. (a) By writing $\tanh x$ in terms of exponentials, show that

$$\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \quad (3)$$

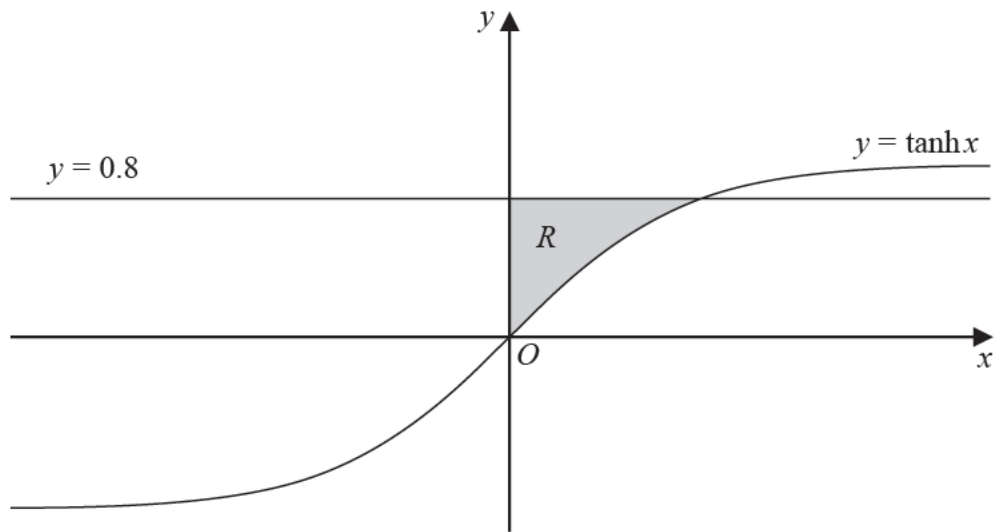


Figure 4

Figure 4 shows a sketch of part of the curve with equation $y = \tanh x$ and the line with equation $y = 0.8$

The finite region R , shown shaded in Figure 4, is bounded by the curve, the line with equation $y = 0.8$ and the y -axis.

The region R is rotated through 2π radians about the x -axis to form a solid of revolution.

(b) Determine the exact value of the volume of this solid of revolution, giving your answer in the form $\pi(a - b \ln 3)$, where a and b are simplified constants to be found.

(7)