8.

$$\mathbf{M} = \begin{pmatrix} 1 & 4 & -1 \\ 3 & 0 & p \\ q & r & s \end{pmatrix}$$

where p, q, r and s are constants and $q > 0$

Given that $\mathbf{M}\mathbf{M}^{\mathrm{T}} = k\mathbf{I}$ for some constant k,

(a) show that $p = 3$

(2)

(b) write down the value of k.

(1)

(c) Hence write down $\mathbf{M}^{-1}$ in terms of q, r and s.

(1)

(d) Determine the exact value of q, the exact value of r and the exact value of s.

(6)