

9. (a) Show that $e^{2i\theta} + e^{-2i\theta} = 2 \cos 2\theta$ (2)

The convergent infinite series C and S are defined as

$$C = 1 + \frac{1}{4} \cos 2\theta + \frac{1}{16} \cos 4\theta + \frac{1}{64} \cos 6\theta + \dots$$

$$S = \frac{1}{4} \sin 2\theta + \frac{1}{16} \sin 4\theta + \frac{1}{64} \sin 6\theta + \dots$$

(b) Show that

$$C + iS = \frac{k}{k - e^{2i\theta}}$$

where k is an integer to be determined.

(3)

(c) Show that

$$C = \frac{16 - 4 \cos 2\theta}{17 - 8 \cos 2\theta}$$

(3)

(d) Hence determine the values of θ , in the range $0 \leq \theta \leq \pi$, for which $C + iS$ is real.

(3)