

Question	Scheme	Marks	AOs
1	$\frac{dy}{dx} = 31 \cosh x - 4 \cosh 2x$	B1	1.1b
	$\frac{dy}{dx} = 31 \cosh x - 4(2 \cosh^2 x - 1)$	M1	3.1a
	$8 \cosh^2 x - 31 \cosh x - 4 = 0$	A1	1.1b
	$(8 \cosh x + 1)(\cosh x - 4) = 0 \Rightarrow \cosh = \dots$	M1	1.1b
	$\cosh x = 4, \left(-\frac{1}{8}\right)$	A1	1.1b
	$\cosh x = \alpha \Rightarrow x = \ln(\alpha + \sqrt{\alpha^2 - 1})$ or $\ln(\alpha + \sqrt{\alpha^2 - 1})$ or $-\ln(\alpha + \sqrt{\alpha^2 - 1})$ or $\ln(\alpha - \sqrt{\alpha^2 - 1})$ or	M1	1.2
	$\frac{e^x + e^{-x}}{2} = 4$ P $e^{2x} - 8e^x + 7 = 0$ P $e^x = \dots$ P $x = \ln(\dots)$		
	$\pm \ln(4 + \sqrt{15})$ or $\ln(4 \pm \sqrt{15})$	A1	2.2a
	(7)		
<b>Alternative</b>			
	$\frac{dy}{dx} = 31 \cosh x - 4 \cosh 2x$ or $31 \left( \frac{e^x + e^{-x}}{2} \right) - 4 \left( \frac{e^{2x} + e^{-2x}}{2} \right)$	B1	1.1b
	Using $\cosh x = \frac{e^x + e^{-x}}{2}$ and $\sinh x = \frac{e^x - e^{-x}}{2}$ as required	M1	3.1a
	P $31 \frac{e^x + e^{-x}}{2} - 4 \frac{e^{2x} + e^{-2x}}{2} = 0$	A1	1.1b
	leading to $4e^{4x} - 31e^{3x} - 31e^x + 4 = 0$ o.e.		
	Solves $4e^{4x} - 31e^{3x} - 31e^x + 4 = 0$ P $e^x = \dots$	M1	1.1b
	$e^x = 4 \pm \sqrt{15}$ or awrt 7.87, 0.13	A1	1.1b
	$x = \ln(b)$ where $b$ is a real exact value	M1	1.2
	$\ln(4 \pm \sqrt{15})$	A1	2.2a
	(7)		

**(7 marks)**

**Notes**

- B1: Correct differentiation
- M1: Identifies a correct approach by using a correct identity to make progress to obtain a quadratic in  $\cosh x$
- A1: Correct 3 term quadratic obtained
- M1: Solves their 3TQ
- A1: Correct values (may only see 4 here)
- M1: Correct process to reach at least one value for  $x$  from their  $\cosh x$

A1: Deduces the correct 2 values with no incorrect values or work involving  $\cosh x = -\frac{1}{8}$

**Alternative**

B1: Correct differentiation

M1: Using the exponential form for  $\cosh x$ , and  $\sinh x$  if required, and forms a quartic equation for  $e^x$  with all terms simplified and all on one side

A1: Correct quartic equation for  $e^x$

M1: Solves their quartic equation in  $e^x$

A1: Correct values to two decimal places or exact values

M1:  $x = \ln(b)$  where  $b$  is a real exact value

A1: Deduces the correct 2 values only