| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 4(a) | $(\cos\theta+\mathrm{i}\sin\theta)^7=\cos^7\theta+\binom{7}{1}\cos^6\theta(\mathrm{i}\sin\theta)+\binom{7}{2}\cos^5\theta(\mathrm{i}\sin\theta)^2+\ldots$<br>Some simplification may be done at this stage<br>e.g. $c^7+7c^6\mathrm{i}s-21c^5s^2-35c^4\mathrm{i}s^3+35c^3s^4+21c^2\mathrm{i}s^5-7cs^6-\mathrm{i}s^7$ | M1 | 1.1b |
| | $\mathrm{i}\sin 7\theta={}^7\mathrm{C}_1c^6\mathrm{i}s+{}^7\mathrm{C}_3c^4\mathrm{i}^3s^3+{}^7\mathrm{C}_5c^2\mathrm{i}^5s^5+\mathrm{i}^7s^7$<br>or $=7c^6\mathrm{i}s+35c^4\mathrm{i}^3s^3+21c^2\mathrm{i}^5s^5+\mathrm{i}^7s^7$ | M1 | 2.1 |
| | $\sin 7\theta=7c^6s-35c^4s^3+21c^2s^5-s^7$ | A1 | 1.1b |
| | $=7(1-s^2)^3s-35(1-s^2)^2s^3+21(1-s^2)s^5-s^7$<br>$=7(1-3s^2+3s^4-s^6)s-35(1-2s^2+s^4)s^3+21(1-s^2)s^5-s^7$ | M1 | 2.1 |
| | $\{7s-21s^3+21s^5-7s^7-35s^3+70s^5-35s^7+21s^5-21s^7-s^7\}$<br>leading to<br>$\sin 7\theta=7\sin\theta-56\sin^3\theta+112\sin^5\theta-64\sin^7\theta$ * | A1* | 1.1b |
| | | **(5)** | |
| (b) | $1+\sin 7\theta=0\Rightarrow\sin 7\theta=-1$ | M1 | 3.1a |
| | $7\theta=-450,-90,270,630,\ldots$<br>or<br>$7\theta=-\frac{5\pi}{2},-\frac{\pi}{2},\frac{3\pi}{2},\frac{7\pi}{2},\ldots$ | A1 | 1.1b |
| | $\theta=-\frac{450}{7},-\frac{90}{7},\frac{270}{7},\frac{630}{7},\ldots\Rightarrow\sin\theta=\ldots$<br>or<br>$\theta=-\frac{5\pi}{14},-\frac{\pi}{14},\frac{3\pi}{14},\frac{7\pi}{14},\ldots\Rightarrow\sin\theta=\ldots$ | M1 | 2.2a |
| | $x=\sin\theta=-0.901,-0.223,0.623,1$ | A1<br>A1 | 1.1b<br>2.3 |
| | | **(5)** | |
| | | | **(10 marks)** |

**Notes**

(a)

M1: Attempts to expand $(\cos\theta+\mathrm{i}\sin\theta)^7$ including a recognisable attempt at binomial coefficients
Some simplification may be done at this stage. (May only see imaginary terms)

M1: Identifies imaginary terms with sin $7\theta$

A1: Correct expression with coefficients evaluated and i's dealt with correctly

M1: Replaces $\cos^2\theta$ with $1-\sin^2\theta$ and applies the expansions of $(1-\sin^2\theta)^2$ and $(1-\sin^2\theta)^3$ to their expression

A1*: Reaches the printed answer with no errors and expansion of brackets seen.

(b)

M1: Makes the connection with part (a) and realises the need to solve $\sin 7\theta=-1$

A1: At least one correct value for $7\theta$

M1: Divides by 7 and deduces that *x* values are found by finding at least one value for $\sin\theta$

A1: Awrt 2 correct values for *x*

A1: Awrt all 4 *x* values correct and no extras