Question	Scheme	Marks	AOs
5(a)	$y=\tan^{-1}x \Rightarrow \tan y = x \Rightarrow \frac{dx}{dy}=\sec^2 y$ $y=\tan^{-1}x \Rightarrow \tan y = x \Rightarrow \frac{dy}{dx}\sec^2 y = 1$	M1	3.1a
	$\frac{dx}{dy}=1+\tan^2 y$ or $\frac{dy}{dx}\left(1+\tan^2 y\right)=1$	M1	1.1b
	$\frac{dy}{dx}=\frac{1}{1+\tan^2 y}=\frac{1}{1+x^2}$ *	A1*	2.1
		(3)	
(b)	$\frac{d\left(\tan^{-1}4x\right)}{dx}=\frac{4}{1+16x^2}$	B1	1.1b
	$\int x\tan^{-1}4x\,dx=\alpha x^2\tan^{-1}4x-\int \alpha x^2\times\left(\frac{4}{1+16x^2}\right)dx$	M1	2.1
	$\int x\tan^{-1}4x\,dx=\frac{x^2}{2}\tan^{-1}4x-\int\frac{x^2}{2}\times\frac{4}{1+16x^2}dx$	A1	1.1b
	$=...-\frac{1}{8}\int\frac{16x^2+1-1}{1+16x^2}dx=...-\frac{1}{8}\int\left(1-\frac{1}{1+16x^2}\right)dx$ or let $4x=\tan u \Rightarrow \frac{1}{8}\int\frac{\tan^2 u}{1+\tan^2 u}\times\frac{1}{4}\sec^2 u\,du$ $\Rightarrow \frac{1}{32}\int\tan^2 u\,du=\frac{1}{32}\int\sec^2 u-u\,du$	M1	3.1a
	$=\frac{x^2}{2}\tan^{-1}4x-\frac{1}{8}x+\frac{1}{32}\tan^{-1}4x+k$	A1	2.1
		(5)	
(c)	Mean value $=\left(\frac{1}{\frac{\sqrt{3}}{4}-0}\right)\left[\frac{x^2}{2}\tan^{-1}4x-\frac{1}{8}x+\frac{1}{32}\tan^{-1}4x\right]_0^{\frac{\sqrt{3}}{4}}$ $=\frac{4}{\sqrt{3}}\left(\left(\frac{3}{32}\times\frac{\pi}{3}-\frac{1}{8}\times\frac{\sqrt{3}}{4}+\frac{1}{32}\times\frac{\pi}{3}\right)-0\right)$	M1	2.1
	$=\frac{\sqrt{3}}{72}\left(4\pi-3\sqrt{3}\right)$ or $\frac{\sqrt{3}}{18}\pi-\frac{1}{8}$ oe	A1	1.1b
		(2)	
		(10 marks)	

Notes

(a)

M1: Makes progress in establishing the derivative by taking the tan of both sides and differentiating with respect to *y* or implicitly with respect to *x*

M1: Use of the correct identity

A1*: Fully correct proof

(b)

B1: Correct derivative

M1: Uses integration by parts in the correct direction

A1: Correct expression

M1: Adopts a correct strategy for the integration by splitting into two fractions or using a substitution of $4x = \tan u$ to get to an integrable form

A1: Correct answer

(c)

M1: Correctly applies the method for the mean value for their integration. The limit of zero can be implied if it comes to 0.

A1: Correct exact answer. Allow exact equivalents e.g. $\frac{4\pi\sqrt{3}-9}{72}$, $\frac{\pi\sqrt{3}}{18}-\frac{1}{8}$