Question	Scheme	Marks	AOs
6(a)	$\lvert\mathbf{M}\rvert = k(-1-1)-5(-1-2)+7(1-2)$ $\{=8-2k\}$	M1	1.1b
	Minors: $\begin{pmatrix}-2 & -3 & -1\\ -12 & -k-14 & k-10\\ -2 & k-7 & k-5\end{pmatrix}$ Cofactors: $\begin{pmatrix}-2 & 3 & -1\\ 12 & -k-14 & 10-k\\ -2 & 7-k & k-5\end{pmatrix}$	M1	1.1b
	$\mathbf{M}^{-1} = \dfrac{1}{8-2k}\begin{pmatrix}-2 & 12 & -2\\ 3 & -k-14 & 7-k\\ -1 & 10-k & k-5\end{pmatrix}$	M1 A1	2.1 1.1b
		(4)	
(b)	$\mathbf{M}^{-1} = \dfrac{1}{4}\begin{pmatrix}-2 & 12 & -2\\ 3 & -16 & 5\\ -1 & 8 & -3\end{pmatrix} \Rightarrow \begin{pmatrix}x\\ y\\ z\end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix}1\\ p\\ 2\end{pmatrix}$ Solve the equations simultaneously to achieve values for x, y and z $y+3z=2p-2$ and $4y+8z=-1 \Rightarrow x=\dots,\ y=\dots,\ z=\dots$	M1	3.1a
	$\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}-\frac{1}{2}+3p-1\\ \frac{3}{4}-4p+\frac{5}{2}\\ -\frac{1}{4}+2p-\frac{3}{2}\end{pmatrix}$	A1ft	1.1b
	$\left(\dfrac{12p-6}{4}, \dfrac{13-16p}{4}, \dfrac{8p-7}{4}\right)$ $\left(3p-\dfrac{3}{2}, \dfrac{13}{4}-4p, 2p-\dfrac{7}{4}\right)$	A1	2.2a
		(3)	
(c)(i)			
	For consistency: E.g. eliminates z to find two equations from $3x+2y=q+2$ $3x+2y=7q-1$ $18x+12y=15$ For consistency: E.g. eliminates x to find two equations from $y+3z=1-4q$ $3y+9z=-3$ $y+3z=2q-2$ For consistency: E.g. eliminates y to find two equations from $x-2z=2-q$ $-x+2z=1-5q$ $-6x+12z=-9$	M1	3.1a
	e.g. $q+2=7q-1 \Rightarrow q=\dots$ e.g. $-3=3(1-4q) \Rightarrow q=\dots$ e.g. $-9=6(1-5q) \Rightarrow q=\dots$	M1	1.1b
	$q=\dfrac{1}{2}$	A1	1.1b

	Alternative Equating coefficients leading to two out of three equations and solves to find values for a and b $4a+b=2,\ 5a+b=1,\ 7a+b=-1$ $\{a=-1, b=6\}$	M1	3.1a
	Forms the fourth equation involving q $a+bq=2$ and substitutes in the values of a and b to finds a value for q	M1	1.1b
	$q=\dfrac{1}{2}$	A1	1.1b
	Finds a coordinate of intersection of the planes $4x+5y+7z=1$ and $2x+y-z=2$ e.g let $z=0 \Rightarrow 4x+5y=1$ and $2x+y=2 \Rightarrow y=-1, x=1.5$	M1	3.1a
	Substitutes the values for x, y and z into $x+y+z=q$ to reach a value for q	M1	1.1b
	$q=\dfrac{1}{2}$	A1	1.1b
(ii)	For example: $x=\lambda \Rightarrow 3\lambda+2y=\dfrac{5}{2},\ \lambda-2z=\dfrac{3}{2} \Rightarrow y=\mathrm{f}(\lambda), z=\mathrm{f}(\lambda)$ $y=\lambda \Rightarrow 3x+2\lambda=\dfrac{5}{2},\ \lambda+3z=1 \Rightarrow x=\mathrm{f}(\lambda), z=\mathrm{f}(\lambda)$ $z=\lambda \Rightarrow 3y+9\lambda=-3,\ -6x+12\lambda=-9 \Rightarrow x=\mathrm{f}(\lambda), y=\mathrm{f}(\lambda)$	M1	3.1a
	Let $x=\lambda,\ \lambda=\dfrac{y-\frac{5}{4}}{-\frac{3}{2}}=\dfrac{z+\frac{3}{4}}{\frac{1}{2}}$ or $y=\dfrac{5}{4}-\dfrac{3}{2}\lambda,\ z=-\dfrac{3}{4}+\dfrac{1}{2}\lambda$ Let $y=\lambda,\ \lambda=\dfrac{x-\frac{5}{6}}{-\frac{2}{3}}=\dfrac{z+\frac{1}{3}}{-\frac{1}{3}}$ or $x=\dfrac{5}{5}-\dfrac{2}{3}\lambda,\ z=-\dfrac{1}{3}-\dfrac{1}{3}\lambda$ Let $z=\lambda,\ \lambda=\dfrac{x-\frac{3}{2}}{2}=\dfrac{y+1}{-3}$ or $x=\dfrac{3}{2}+2\lambda,\ y=-1-3\lambda$	A1	1.1b
	$\mathbf{r}=\dfrac{5}{4}\mathbf{j}-\dfrac{3}{4}\mathbf{k}+t(2\mathbf{i}-3\mathbf{j}+\mathbf{k})$ o.e. $\mathbf{r}=\dfrac{5}{6}\mathbf{i}-\dfrac{1}{3}\mathbf{k}+t(2\mathbf{i}-3\mathbf{j}+\mathbf{k})$ o.e. $\mathbf{r}=\dfrac{3}{2}\mathbf{i}-\mathbf{j}+t(2\mathbf{i}-3\mathbf{j}+\mathbf{k})$ o.e.	M1 A1	1.1b 2.5
	Alternative (ii) Finds two different coordinates that lie on the line of intersection For example: setting $x=0 \Rightarrow \left(0, \dfrac{5}{4}, -\dfrac{3}{4}\right)$ setting $y=0 \Rightarrow \left(\dfrac{5}{6}, 0, -\dfrac{1}{3}\right)$ setting $z=0 \Rightarrow \left(\dfrac{3}{2}, -1, 0\right)$	M1 A1	3.1a 1.1b
	Finds the vector equation of the line passing through their two points	M1	1.1b
	$\mathbf{r}=\dfrac{5}{4}\mathbf{j}-\dfrac{3}{4}\mathbf{k}+t(2\mathbf{i}-3\mathbf{j}+\mathbf{k})$ o.e.	A1	2.5

	$\mathbf{r}=\frac{5}{6}\mathbf{i}-\frac{1}{3}\mathbf{k}+t(2\mathbf{i}-3\mathbf{j}+\mathbf{k})$ o.e. $\mathbf{r}=\frac{3}{2}\mathbf{i}-\mathbf{j}+t(2\mathbf{i}-3\mathbf{j}+\mathbf{k})$ o.e.		
	Alternative (ii): Outside the spec Finds the cross product of two normal vectors and a coordinate that lies on all three planes	M1	3.1a
	Correct cross product $\begin{vmatrix}4 & 5 & 7\\1 & 1 & 1\end{vmatrix}=\begin{vmatrix}4 & 5 & 7\\2 & 1 & -1\end{vmatrix}=\begin{pmatrix}2\\3\\-1\end{pmatrix}$ or $\begin{vmatrix}1 & 1 & 1\\2 & 1 & -1\end{vmatrix}=\begin{pmatrix}12\\18\\-3\end{pmatrix}$	A1	1.1b
	Uses the point and the direction vector to find the equation of the line	M1	1.1b
	$\mathbf{r}=\frac{5}{4}\mathbf{j}-\frac{3}{4}\mathbf{k}+t(2\mathbf{i}-3\mathbf{j}+\mathbf{k})$ o.e.	A1	1.1b
		(7)	
			(14 marks)

Notes

(a)

M1: Correct method to find the determinant. Condone one sign slip

M1: A correct first step in obtaining the inverse. Could be the matrix of minors or cofactors. Condone sign slips as long as the intention is clear.

M1: Fully correct method to obtain the inverse. Attempts matrix of minors, cofactors, transposes and 1/determinant

A1: Correct matrix

(b)

M1: A complete strategy for solving the given equations e.g. multiplies the given coordinates by their inverse or solves simultaneously to achieve values for *x*, *y* and *z*

A1ft: Correct calculation on their inverse matrix (unsimplified) or at least on correct value if solving simultaneously

A1: Correct coordinates

(c)(i)

M1: Uses a correct strategy that will lead to establishing a value for *q*. E.g. eliminating one of *x*, *y* or *z*

M1: Solves a suitable equation to obtain a value for *q*

A1: Correct value

(c)(i) **Alternative 1**

M1: Equating coefficients leading to two out of three equations and solves to find values for a and b

M1: Solves a suitable equation to obtain a value for *q* using their values for *a* and *b*

(c)(i) **Alternative 2**

M1: Finds a coordinate of intersection of the planes $4x+5y+7z=1$ and $2x+y-z=2$

M1: Substitutes the values for *x*, *y* and *z* into $x+y+z=q$ to reach a value for *q*

A1: Correct value

(ii)

M1: Uses a correct strategy to obtain the Cartesian equation of the line or the general coordinates

A1: Correct Cartesian equation or coordinates in terms of a parameter.

M1: Uses their Cartesian equation to correctly extract the position and direction to form a vector equation for the required line
A1: Correct equation (o.e.) look out for multiples of the direction vector

Alternative (ii)

M1: Finds two different coordinates that lie on the line of intersection
A1: Correct coordinates
M1: Uses their coordinates to find the vector equation of the line that passes through them.
A1: Correct equation (o.e.), look out for multiples of the direction vector. Must have $\mathbf{r} = \ldots$

Alternative (ii) outside spec

M1: Finds the cross product between the normal vectors of two of the planes and a coordinate that lies on all three planes. If a coordinate is found in (i) it must be used in this part to award this mark.
A1: Correct cross product
M1: Uses the coordinate and the cross product to find the equation of the line
A1: Correct equation (o.e.), look out for multiples of the direction vector. Must have $\mathbf{r} = \ldots$